A Relational Framework for Information Extraction

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ABSTRACT

Information Extraction commonly refers to the task of populating a relational schema, having predefined underlying semantics, from textual content. This task is pervasive in contemporary computational challenges associated with Big Data. In this article we provide an overview of our work on document spanners—a relational framework for Information Extraction that is inspired by rule-based systems such as IBM’s SystemT.

Categories and Subject Descriptors

F.4.3 [Mathematical Logic and Formal Languages]: Formal Languages—Algebraic language theory, Classes defined by grammars or automata, Operations on languages; F.1.1 [Computation by Abstract Devices]: Models of Computation—Automata, Relations between models; H.2.4 [Database Management]: Systems—Textual databases; I.5.4 [Pattern Recognition]: Applications—Text processing

General Terms

Theory

Keywords

Information extraction, document spanners, regular expressions, automata, inconsistency, prioritized repairs

1. INTRODUCTION

Information Extraction (IE) refers to the task of discovering structured information in textual content. More precisely, the goal in IE is to populate a predefined relational schema that has predetermined underlying semantics, by correctly detecting the values of records in a given text document or a collection of text documents. Popular tasks in the space of IE include named entity recognition [29] (identify proper names in text, and classify those into a predefined set of categories such as person and organization), relation extraction [34] (extract tuples of entities that satisfy a predefined relationship, such as person-organization), event extraction [3] (find events of predefined types along with their key players, such as nomination and nominee), temporal information extraction [15,25] (associate mentions of facts with mentions of their validity period, such as nomination-date), and coreference resolution [27] (match between phrases that refer to the same entity, such as “Obama,” “the President,” and “him”).

As a discipline, IE began with the DARPA Message Understanding Conference (MUC) in 1987 [22]. While early work in the area focused largely on military applications, this task is nowadays pervasive in a plethora of computational challenges, in particular those associated with Big Data, such as social media analysis [6], machine data analysis [21], healthcare analysis [33], semantic search [35], and customer relationship management [2]. In a typical text-analytics pipeline (e.g., [32]), the output of IE is fed into a cleaning and/or fusion component, such as an entity-resolution algorithm, that in turn produces input for a global processing phase (e.g., statistical analysis or data mining). Contemporary business models like cloud computing, along with analytics platforms like Hadoop, facilitate such data analyses for a broad range of individuals and organizations.

Most information extraction systems incorporate a notion of rules in a domain-specific rule language. These rules may define the entire extraction task, or produce features for downstream statistical models. The rules may be manually coded, or automatically learned. The choice of a rule language comprises an important part of an IE system’s design. Designing such a language involves navigating a number of tradeoffs, with the most important of these being that of simplicity versus expressivity. Keeping a rule language simple pays dividends in multiple ways. A simple rule language, with relatively few language constructs and a straightforward semantics, is easier for users to understand and debug, easier for learning algorithms to train, and easier for a rule engine to execute with high throughput. But such simplicity can easily compromise the expressiveness of...
the rule language, and thus lower the quality of extrac-
tion results. Limited expressiveness of a rule language
can force developers to augment the rules with custom
code in a general-purpose language like Java or Python.
This practice, though often necessary to achieve accept-
able accuracy, makes development, maintenance, and
performance tuning significantly more difficult.

This article describes our recent work on a formal
framework for examining the expressivity of IE rule lan-
guages. The framework, called document spanners, lever-
ages known principles of database management. The
framework itself is introduced in Sections 2 and 3. In
Section 4 we give results on expressiveness, and in Sec-
tion 5 we discuss conflict resolution within the frame-
work. We discuss the impact of the string-equality op-
erator in Section 6, and conclude in Section 7.

2. DOCUMENT SPANNERS

In this section, we give some preliminary definitions
and recall the formalism of document spanners [19].

We fix a finite alphabet Σ of symbols. We denote by
Σ* the set of all finite strings over Σ, and by Σ+ the
set of all finite strings of length at least one over Σ.
For clarity of context, we will often refer to a string in Σ* as
a document. 1 A language over Σ is a subset of Σ*. Let
d = σ1 · · · σn ∈ Σ* be a document. The length n of d
is denoted by |d|. A span identifies a substring of d by
specifying its bounding indices. Formally, a span of d
has the form [i, j], where 1 ≤ i ≤ j ≤ n + 1. If [i, j] is a span
of d, then d[i, j) denotes the substring σi · · · σj−1. Note
that d[i, i) is the empty string, and that d[i, n + 1)
is d. The more standard notation would be [i, j), but
we use [i, j) to distinguish spans from intervals. For example,
[1, 1) and [2, 2) are both the empty interval, hence equal,
but in the case of spans we have [1, 1) = [1′, 1′) if and only if
i = i′ and j = j′ (and in particular, [1, 1) ̸= [2, 2)). We denote by Spans(d) the set of all
the spans of d. Two spans [i, j) and [i′, j′) of d are
disjoint if j ≤ i′ or j′ ≤ i, and they overlap otherwise.
Finally, [i, j) contains [i′, j′) if i ≤ i′ ≤ j′ ≤ j.

EXAMPLE 2.1. In all of the examples throughout the
article, we consider the example alphabet Σ which con-
sists of the lowercase and capital letters from the English
alphabet (i.e., a, . . . , z and A, . . . , Z), the comma symbol
(“,”), and the underscore symbol (“_”) that stands for
whitespace. (We use a restricted alphabet for simplic-
ty.) Figure 1 depicts an example document d in Σ*.
For ease of later reference, it also depicts the index of
each character in d. Figure 2 shows two tables contain-
ing spans of d. Observe that the spans in the left table of
Figure 2 are those that correspond to words in d that are
names of US states (Georgia, Washington and Vir-
inia). For example, the span [21, 28) corresponds to
Georgia. We will further discuss the meaning of these
tables later.

We fix an infinite set SVars of (span) variables; spans
may be assigned to these variables. The sets Σ* and
SVars are disjoint. For a finite set V ⊆ SVars of vari-
bles and a document d ∈ Σ*, a (V, d)-tuple is a mapping
µ: V → Spans(d) that assigns a span of d to each variable in V. A (V, d)-relation is a set of (V, d)-
tuples. A document spanner (or just spanner for short)
is a function P that is associated with a finite set V of
variables, denoted SVars(P), and that maps every doc-
ument d to a (V, d)-relation.

EXAMPLE 2.2. Throughout the article we will define
several spanners. Two of those are denoted as [[µun]] and
[[µoe]], where SVars([[µun]]) = {x} and SVars([[µoe]]) = {x1, x2, y}. Later we will explain the meaning of the
[ ] brackets, and specify what exactly each spanner ex-
tracts from a given document. For now, the span rela-
tions (tables) in Figure 2 show the results of applying
the two spanners to the document d of Figure 1.

Let P be a spanner with SVars(P) = V. Let d ∈ Σ* be a
document, and let µ ∈ P(d) be a (V, d)-tuple. We say that µ is hierarchical if for all variables x, y ∈ SVars(P) one of the following holds: (1) the span µ(x) contains µ(y), (2) the span µ(y) contains µ(x), or (3) the spans µ(x) and µ(y) are disjoint. As an example, the reader can verify that all the tuples in Figure 2 are hierarchical. We say that P is hierarchical if µ is hier-
archical for all d ∈ Σ* and µ ∈ P(d). Observe that for two variables x and y of a hierarchical spanner, it
may be the case that, over the same document, one tu-
ple maps x to a subspan of y, another tuple maps y to a
subspan of x, and a third tuple maps x and y to disjoint
spans. Finally, we say that P is Boolean if SVars(P) is
empty. Note that when P is Boolean, its application to
a string d is either the empty set (false) or the singleton
that consists of the empty tuple (true).

3. SPANNER REPRESENTATION

By a spanner representation system we refer collec-
tively to any manner of specifying spanners through fi-
nite objects. In this section we recall several representa-
tion systems that we have proposed and studied in pre-
vious work [18, 19]: regex formulas, spanner algebra,
basic extraction programs, and automata.

3.1 Regex Formulas

Regular expressions are one of the oldest types of in-
formation extraction rule languages. Many of the sys-
tems deployed in early MUC competitions used regular
expressions over characters or token streams as their primary language. A more recent example of a system with a regular-expression-based rule language is the JAPE system [14], in which rules comprise regular expressions over streams of tokens, and rule evaluation is via a finite-state transducer.

A regular expression with capture variables, or just variable regex for short, is an expression in the following syntax that extends that of regular expressions:

\[ \gamma = \emptyset | \varepsilon | \sigma | \gamma \lor \gamma | \gamma \cdot \gamma | \gamma^* | x(\gamma) \] (1)

The added alternative is \( x(\gamma) \), where \( x \in SVars \). We denote by SVars(\( \gamma \)) the set of variables that occur in \( \gamma \). We use \( \gamma^+ \) as abbreviations of \( \gamma \cdot \gamma^* \).

A variable regex can be matched against a document in multiple ways, or more formally, there can be multiple parse trees showing that a document matches a variable regex. Each parse tree associates variables with spans. It is possible, however, that in a parse tree a variable is not associated with any span, or is associated with multiple spans. If every variable is associated with precisely one span, then the parse tree is said to be functional. A variable regex is called a regex formula if it has only functional parse trees on every input document. An example of a variable regex that is not a regex formula is \((x[a])^*\), because a match against \(aa\) assigns \( x \) to two spans. We refer to Fagin et al. [19] for the full formal definition of regex formulas. By RGX we denote the class of regex formulas. A regex formula \( \gamma \) is naturally viewed as representing a spanner, and by \( \gamma \) we denote the spanner that is represented by \( \gamma \). Following are examples of spanners represented as regex formulas.

**EXAMPLE 3.1.** In the regex formulas of our running examples we will use the following conventions.

- \([a-z]\) denotes the disjunction \( a \lor \cdots \lor z \);
- \([A-Z]\) denotes the disjunction \( A \lor \cdots \lor Z \);
- \([a-zA-Z]\) denotes \( [a-z] \lor [A-Z] \);
- \( \Sigma \), by abuse of notation, denotes the regex formula recognizing all symbols in \( \Sigma \), i.e., \( \Sigma \) denotes the disjunction \( [a-zA-Z] \lor \cdots \lor \).

We now define several regex formulas that we will use throughout the article.

The following regex formula extracts tokens (which for our purposes now are simply complete words) from text. (Note that this is a simplistic extraction for the sake of presentation.)

\[ \gamma_{ten} \overset{\text{df}}{=} (\varepsilon \lor (\Sigma^* \cdot \_ \_ \_ \_ \_)) \cdot x([a-zA-Z]^+) \cdot ((\_ \lor \_ \_ \lor \Sigma^*) \lor \varepsilon) \]

When applied to the document \( d \) of Figure 1, the resulting spans include \([1, 7], [8, 12], [13, 19]\) and so on.

The following regex formula extracts spans that begin with a capital letter.

\[ \gamma_{icap} \overset{\text{df}}{=} \Sigma^+ \cdot x([A-Z] \cdot \Sigma^*) \cdot \Sigma^* \]

When applied to the document \( d \) of Figure 1, the resulting spans include \([1, 7], [1, 3], [13, 19] \), and so on.

The following regex formula extracts all the spans that span names of US states. For simplicity, we include just the three in Figure 1. For readability, we omit the concatenation symbol - between two alphabet symbols.

\[ \gamma_{statt} \overset{\text{df}}{=} \Sigma^+ \cdot x\{\text{Georgia} \lor \text{Virginia} \lor \text{Washington}\} \cdot \Sigma^* \]

When applied to the document \( d \) of Figure 1, the resulting spans are \([28, 30], [40, 40], \) and \([60, 68] \).

The following regex formula extracts all the triples \((x_1, x_2, y)\) of spans such that the string "_ _ _" separates \( x_1 \) and \( x_2 \), and \( y \) is the span that starts where \( x_1 \) starts and ends where \( x_2 \) ends.

\[ \gamma_{\_ _ _} \overset{\text{df}}{=} \Sigma^+ \cdot y\{x_1(\Sigma^*) \cdot \_ \_ \cdot x_2(\Sigma^*)\} \cdot \Sigma^+ \]

Let \( d \) be the document of Figure 1, and let \( V \) be the set \( \{x_1, x_2, y\} \) of variables. The \((V, d)\)-tuples that are obtained by applying \( \gamma \) to \( d \) map \((x_1, x_2, y)\) to triples like \([(13, 19), [21, 28], [13, 28]]\), and in addition, triples that do not necessarily consist of full tokens, such as the triple \([(9, 19), [21, 23], [9, 23]]\).

3.2 Algebra over Spanners

Some IE systems use rule languages whose semantics derive from the relational calculus. For example, the Xlog system [28] system has a Datalog-based rule language, while SystemT [10] has a rule language based on SQL. These systems use rule engines that combine the relational algebra with automata for evaluating character-level primitives such as regular expressions. Such an algebraic runtime allows for efficient rule execution via query optimization. We can model this class of execution.

<table>
<thead>
<tr>
<th>(<a href="d">\rho_{sm}</a>)</th>
<th>(<a href="d">\rho_{fnz}</a>)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(x_1, x_2, y)</td>
</tr>
<tr>
<td>(\mu_1)</td>
<td>([21, 28])</td>
</tr>
<tr>
<td>(\mu_2)</td>
<td>([30, 40])</td>
</tr>
<tr>
<td>(\mu_3)</td>
<td>([60, 68])</td>
</tr>
</tbody>
</table>

**Figure 2:** Results of spanners in the running example
tion environment by extending regex formulas with a relational algebra.

Let \( \mathcal{R} \) be a representation system for spanners. Given a collection \( O \) of relational algebraic operators, we denote by \( \mathcal{R}^O \) the closure of \( \mathcal{R} \) under the operators of \( O \). Here relational operators are extended pointwise to spanners. For example, consider \( O = \{ \bowtie, \star \} \), where \( \bowtie \) is the natural join operator. Then \( \mathcal{R}^O \) consists of all spanners in \( \mathcal{R} \), along with, for all spanners \( P_1 \) and \( P_2 \) definable in \( \mathcal{R} \), the spanner \( [P_1 \bowtie P_2] \), which is defined by \([P_1 \bowtie P_2](d) = P_1(d) \bowtie P_2(d)\) for all documents \( d \).

Note in particular that the natural join here is based on span equality, not on string equality, since our relations contain spans.

We consider here three operators of positive relational algebra: union (\( \cup \)), projection (\( \pi \)), and natural join (\( \bowtie \)). Observe that the projection operator is parameterized by a sequence of variables from it operand spanner; that is, the operator has the form \( \pi_{x_1, \ldots, x_n} \) where \( x \) is a sequence of variables. The standard typing rules for union and projection apply: union can only be applied to spanners \( P_1 \) and \( P_2 \) if \( \text{SVars}(P_1) = \text{SVars}(P_2) \), and \( \pi_x \) is only applicable to spanner \( P \) if every member of \( x \) is in \( \text{SVars}(P) \). As usual, by \( [\rho] \) we denote the spanner that is represented by the algebraic expression \( \rho \).

In the next example, we use the following notation. Let \( \rho \) be an expression in an algebra over \( \text{RGX}(\cup, \pi, \bowtie) \) and let \( x = x_1, \ldots, x_n \) be a sequence of \( n \) distinct variables containing all the variables in \( \text{SVars}(\rho) \) (and possibly additional variables). Let \( y = y_1, \ldots, y_n \) be a sequence of distinct variables of the same length as \( x \). We denote by \( \rho[y/x] \) the expression \( \rho' \) that is obtained from \( \rho \) by replacing every occurrence of \( x_i \) with \( y_i \). If \( x \) is clear from the context, then we may write just \( \rho[y] \).

**Example 3.2.** Using the regex formulas from Example 3.1, we define several \( \text{RGX}(\cup, \pi, \bowtie) \)-spanners.

- The spanner \( \rho_{\text{str}} = \gamma_{\text{str}} \bowtie \gamma_{\text{str}} \) extracts all the tokens that are names of US states. Note that, since \( \text{SVars}(\gamma_{\text{str}}) = \gamma_{\text{str}} = \{ x \} \), the natural join actually computes an intersection.

- The spanner \( \rho_{\text{cap}} = \gamma_{\text{cap}} \bowtie \gamma_{\text{cap}} \) extracts all the tokens beginning with a capital letter.

- The spanner \( \rho_{\text{loc}} = \rho_{\text{cap}}[x_1/x] \bowtie \rho_{\text{str}}[x_2/x] \bowtie \gamma_{\text{loc}} \) extracts spans of strings including “city, state.”

The results of applying the spanners \( [\rho_{\text{str}}] \) and \( [\rho_{\text{loc}}] \) to the document \( d \) of Figure 1 are in Figure 2. Note that the right column of the right table in the figure is obtained through the spanner \( \pi_{x_1}(\rho_{\text{loc}}) \), and the union of the left and middle columns is obtained through the spanner \( (\pi_{x_1}(\rho_{\text{loc}})) \cup (\pi_{x_2}(\rho_{\text{loc}})) \).

Later on, we will discuss several additional operators, including selection, difference and complement.

<table>
<thead>
<tr>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( f_4 )</th>
<th>( f_5 )</th>
<th>( f_6 )</th>
<th>( f_7 )</th>
<th>( f_8 )</th>
<th>( f_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>([13, 28])</td>
<td>([21, 40])</td>
<td>([46, 68])</td>
<td>([1, 7])</td>
<td>([13, 19])</td>
<td>([21, 28])</td>
<td>([30, 40])</td>
<td>([46, 68])</td>
<td>([60, 68])</td>
</tr>
</tbody>
</table>

**Figure 3:** A d-instance \( I \) over the signature of the running example

### 3.3 Basic Extraction Programs

In [18], we used the Datalog syntax for specifying spanners. We describe a basic form of this syntax (which we later extend) in this section.

A signature is a finite sequence \( S = \langle R_1, \ldots, R_m \rangle \) of distinct relation symbols, where each \( R_i \) has an arity \( a_i > 0 \). In this work, the data is a document \( d \), and entries in the instances of a signature are spans of \( d \). Formally, for a signature \( S = \langle R_1, \ldots, R_m \rangle \) and a document \( d \in \Sigma^* \), a d-instance (over \( S \)) is a sequence \( \langle r_1, \ldots, r_m \rangle \), where each \( r_i \) is a relation of arity \( a_i \) over \( \text{Spans}(d) \); that is, \( r_i \) is a subset of \( \text{Spans}(d)^{a_i} \). A d-fact (over \( S \)) is an expression of the form \( R(s_1, \ldots, s_n) \), where \( R \) is a relation symbol of \( S \) with arity \( a \), and each \( s_i \) is a span of \( d \). If \( f \) is a d-fact \( R(s_1, \ldots, s_n) \) and \( I \) is a d-instance, both over the signature \( S \), then we say that \( f \) is a fact of \( I \) if \( (s_1, \ldots, s_n) \) is a tuple in the relation of \( I \) that corresponds to \( R \). For convenience of notation, we identify a d-instance with the set of its facts.

**Example 3.3.** The signature \( S \) for our running example consists of three relation symbols:

- The unary relation symbol \( \text{Loc} \) stands for location;
- The unary relation symbol \( \text{Per} \) stands for person;
- The binary relation symbol \( \text{PerLoc} \) associates persons with locations.

We continue with our running example. Figure 3 shows a d-instance over \( S \), where \( d \) is the document of Figure 1. This instance has 12 facts, and for later reference we denote them by \( f_1, \ldots, f_{12} \). Note that there are quite a few mistakes in the table (e.g., the annotation of Virginia as a person by fact \( f_9 \)); in the next section we will show how these are dealt with in the framework of this article.

Let \( \mathcal{R} \) be a spanner representation system. A basic extraction program in \( \mathcal{R} \), or just basic \( \mathcal{R} \)-program, for short, is a triple \( (S, U, \phi) \), where \( S \) is a signature, \( U \) is a finite sequence \( u_1, \ldots, u_m \) of Horn rules, and \( \phi \) is an atomic formula over \( S \) (representing the result of the program). Here, an atomic formula \( \phi \) is an expression of the form \( \text{R}(x_1, \ldots, x_n) \), where \( \text{R} \) is an \( n \)-ary relation symbol in \( S \). A Horn rule has the form \( \text{R}(y_1, \ldots, y_a) :- \)
one of the following holds for cables on $x$. We denote by $\langle R \rangle$ the class of basic $R$-programs.

**Example 3.4.** We now define a basic $\text{RGX}^{(\omega,\pi,\infty)}$-program $\mathcal{E}$ in our running example. Intuitively, the goal of the program is to extract pairs $(x, y)$, where $x$ is a person and $y$ is a location associated with $x$.

The signature is that of Example 3.3. The sequence $U$ of rules is the following. Note that we are using the notation we established in the previous examples.

1. $\text{Loc}(x) := \rho_{\text{loc}}[x]$ (see Example 3.2)
2. $\text{Per}(y) := \rho_{\text{cap}}[y]$ (see Example 3.2)
3. $\text{PerLoc}(x,y) := \text{Per}(x) \land \text{Loc}(y) \land \text{precede}[x,y]

In the above program, precede is the regex formula $\Sigma^* \cdot x[\Sigma^*] \cdot \Sigma^* \cdot y[\Sigma^*] \cdot \Sigma^*$. Hence, precede states that $x$ terminates before $y$ begins.

### 3.4 Automata

Next, we recall a representation by means of automata. A **variable-set automaton** (or **vset-automaton**) is a tuple $(Q, q_0, q_f, \delta)$, where: $Q$ is a finite set of states, $q_0 \in Q$ is an initial state, $q_f \in Q$ is an accepting state, and $\delta$ is a finite transition relation consisting of triples, each having one of the forms $(q, \sigma, q')$, $(q, \epsilon, q')$, $(q, x \rightarrow q')$, or $(q, \neg x, q')$, where $q, q' \in Q$, $\sigma \in \Sigma$, and $x \in \text{SVars}$. We denote by $\text{SVars}(A)$ the set of variables that occur in the transitions of $A$.

Let $d = \sigma_1 \cdots \sigma_n$ be a document. A configuration of a vset-automaton $A = (Q, q_0, q_f, \delta)$, when running on $d$, is a tuple $c = (q, V, Y, i)$, where $q \in Q$ is the current state, $V \subseteq \text{SVars}(A)$ is the set of active variables, $Y \subseteq \text{SVars}(A)$ is the set of available variables, and $i$ is an index in $\{1, \ldots, n + 1\}$. A run $c$ of $A$ on $d$ is a sequence $c_0, \ldots, c_m$ of configurations, where $c_0 = (q_0, V_0, Y_0, 1)$, and for $j = 0, \ldots, m - 1$ one of the following holds for $c_j = (q_j, V_j, Y_j, i_j)$ and $c_{j+1} = (q_{j+1}, V_{j+1}, Y_{j+1}, i_{j+1})$:

1. $V_{j+1} = V_j$, $Y_{j+1} = Y_j$, and either (a) $i_{j+1} = i_j + 1$ and $(q_j, \delta, q_{j+1}) \in \delta$ (ordinary transition), or (b) $i_{j+1} = i_j$ and $(q_j, \epsilon, q_{j+1}) \in \delta$ (epsilon transition).
2. $i_{j+1} = i_j$ and for some $x \in \text{SVars}(A)$, either (a) $x \in Y_j$, $V_{j+1} = V_j \cup \{x\}$, $Y_{j+1} = Y_j \setminus \{x\}$, and we have $(q_j, \neg x, q_{j+1}) \in \delta$ (variable insert), or (b) $x \in V_j$, $V_{j+1} = V_j \setminus \{x\}$, $Y_{j+1} = Y_j$ and $(q_j, \epsilon, q_{j+1}) \in \delta$ (variable remove).

Note that in a run, each configuration $(q, V, Y, i)$ is such that $V$ and $Y$ are disjoint. The run $c = c_0, \ldots, c_m$ is $\alpha_1 \land \cdots \land \alpha_k$, where $R$ is a relation symbol of $\Sigma$ of arity $a$, and each $\alpha_i$ is either an atomic formula over $\Sigma$ or a spanner in $R$.

We make the requirement that each $y_j$ occurs in at least one $\alpha_i$. We denote by $\text{BPR}\langle R \rangle$ the class of basic $R$-programs.

### 4. Regular Spanners and Expressiveness

We now give results on the expressiveness of the representation systems of the previous section. Given a representation system $R$, we denote by $[R]$ the class of spanners definable by $R$. The following theorem shows that several of the representation systems defined in the previous section have the same expressive power.

**Theorem 4.1.** [18,19] The following representation systems have precisely the same expressive power.

- The closure of regex formulas under union, projection and natural join.
- The basic RGX-programs.
- The vset-automata.

*That is, $[\text{RGX}^{(\omega,\pi,\infty)}] = [\text{BPR}\langle \text{RGX} \rangle] = [\text{VA}_{\text{set}}].$*

A spanner is regular if it is definable in the representation systems of Theorem 4.1. We denote by $\text{REG}$ the set of expressions in $\text{RGX}^{(\omega,\pi,\infty)}$. Hence, all of the representation systems of the theorem capture precisely
Note, however, that regex formulas are strictly less expressive than regular spanners. This is true, since a spanner defined by a regex formula is necessarily hierarchical. The following theorem shows that regex formulas capture precisely those regular spanners that are hierarchical.

**Theorem 4.2.** [19] A spanner $P$ is definable in RGX if and only if $P$ is both regular and hierarchical.

Next, we discuss the selection operator. Let $R$ be a $k$-ary string relation, and let $P$ be a spanner. The string-selection operator $\zeta^R$ is parameterized by $k$ span variables $x_1, \ldots, x_k$ and may be written as $\zeta^R_{x_1, \ldots, x_k}$. If $P^d$ is $\zeta^R_{x_1, \ldots, x_k} P$, then $P^d(d)$ is the restriction of $P(d)$ to those $d$-tuples $\mu$ such that $(d_{\mu(x_1)}, \ldots, d_{\mu(x_k)}) \in R$. For example, if $R$ is the binary relation consisting of all the pairs of strings that start with the same symbol, then $\zeta^R_{x,y} P(d)$ is obtained from $P(d)$ by removing all the tuples $\gamma$ in which the strings spanned by $\gamma(x)$ and $\gamma(y)$ start with different symbols.

A string relation is a relation over $\Sigma^*$. A $k$-ary string relation $R$ is recognizable [7, 16] if it is a finite union of Cartesian products $L_1 \times \cdots \times L_k$, where each $L_i$ is a regular language over $\Sigma$. We have the following.

**Theorem 4.3.** [19] Let $R$ be a string relation. RGX is closed under the selection operator $\zeta^R$ if and only if $R$ is recognizable.

Finally, we discuss difference and complementation. We denote by $\setminus$ the difference operator, and by $\sim$ the complement operator. Here, the complement of a spanner $P$ is the spanner $Q$ that has the same variables as $P$, and for every document $d$, the tuples in $Q(d)$ are precisely those involving spans of $d$ that are not in $P$. (Note that $Q(d)$ is finite since there only finitely many spans over $d$.) Difference is defined as usual.

**Theorem 4.4.** [19] Regular spanners are closed under difference and complement; that is:

$\text{REG} = \text{REG} \setminus \sim = \text{RGX} \setminus \sim$

5. **CONFLICT RESOLUTION**

It is a common practice for different rules of an IE rule set to match the same region of text in different ways. Allowing this kind of overlap simplifies the task of developing and maintaining the rules if the rules are written by hand; and it simplifies the learning problem in systems that induce rules from examples. Nearly every IE rule system in use today allows for conflicting rules and provides language features for resolving these conflicts. Examples of such language features include the controls in the JAPE rule language [14] and the “consolidate” clause in SystemT’s AQL [10]. The sections that

![Figure 5: A conflict graph with priorities in the running example](image)

follow describe our theoretical framework for a declarative language for specifying policies for conflict resolution [18].

5.1 **Conflicts and Priorities**

Observe that the instance of Figure 3 contains several conflicting facts. For example, $f_2$ represents a location, but it has a nonempty intersection with $f_6$ and $f_7$, which stand for person mentions. The database research community has established the concept of repairs as a mechanism for handling inconsistencies in a declarative fashion [5]. Conventionally, denial constraints are specified to declare sets of facts that cannot co-exist in a consistent instance. A repair of an inconsistent instance is a consistent subinstance that is not properly contained in any other consistent subinstance.

We adapt the concept of denial constraints to our setting. In the world of IE, the repairs are not necessarily all equal. In fact, in every example we are aware of, the developer has a clear preference as to which facts to exclude when a denial constraint is violated. Therefore, instead of the traditional repairs, we will use the notion of prioritized repairs of Staworko et al. [30], which extends repairing by incorporating priorities.

Let $S$ be a signature, let $d$ be a document, and let $I$ be a $d$-instance over $S$. A conflict hypergraph for $I$ is a hypergraph $H$ over the facts of $I$; that is, $H = (V, E)$ where $V$ is the set of $I$’s facts and $E$ is a collection of hyperedges (subsets of $V$). Intuitively, the hyperedges represent sets of facts that together are in conflict. A priority relation for $I$ is a binary relation $\succ$ over the facts of $I$. If $f$ and $f'$ are facts of $I$, then $f \succ f'$ means intuitively that $f$ is preferred to $f'$. A repair of $I$ is a maximal subinstance of $I$ that does not contain any hyperedge of $H$. To accommodate priorities in cleaning, we use the notion of Pareto optimality [30]: a consistent subinstance $J$ is an improvement of a consistent subin-
Figure 6: A d-instance \(J_3\) over the signature of the running example

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Loc} & \text{Per} & \text{PerLoc} \\
\hline
f_1 & (13, 28) & f_4 & (1, 7) & f_{10} & (1, 7) & (13, 28) \\
f_3 & (46, 68) & f_5 & (30, 40) & f_{12} & (30, 40) & (46, 68) \\
\hline
\end{array}
\]

5.2 Denial Constraints and Priority Generating Dependencies

We now discuss the syntactic declaration of conflicts and priorities. To specify a conflict hypergraph at the signature level (i.e., to specify the conflict hypergraph for every instance), we use the formalism of denial constraints. Let \(S\) be a signature, and let \(R\) be a spanner representation system. A denial constraint in \(R\) (over \(S\)), or just \(R\)-dc (or simply dc) for short, has the form

\[
\forall x[P \rightarrow \neg \Psi(x)]
\]

where \(x\) is a sequence of variables in SVars, \(P\) is a spanner specified in \(R\) with all of its variables in \(x\), and \(\Psi\) is a conjunction of atomic formulas over \(S\). We usually omit the universal quantifier, and specify a dc simply by \(P \rightarrow \neg \Psi(x)\). Semantically, \(P \rightarrow \neg \Psi(x)\) is interpreted in the usual first-order-logic sense while viewing \(P\) as a predicate that contains all of the tuples in its output; that is, \(P \rightarrow \neg \Psi(x)\) is satisfied in a document \(d\) if for every \((x, d)\)-tuple \(\mu\), if \(P(d)\) contains the restriction of \(\mu\) to SVars\((P)\), then at least one of the conjuncts of \(\Psi\) must be false under \(\mu\).

Example 5.2. We now define dcs in our running example. Recall that precede is a regex formula stating that \(x\) terminates before \(y\) begins. We denote by disjoint the regex formula \(\operatorname{precede}(x, y) \lor \operatorname{precede}(y, x)\). We denote by overlap an expression in REG that represents the complement of disjoint. Note that overlap is indeed expressible by a regular spanner, since regular spanners are closed under complement (Theorem 4.4). Finally, we denote by \(\text{overlap}_{d}\) an expression in REG that restricts the pairs in overlap to those \((x, y)\) satisfying \(x \neq y\) (i.e., \(x\) and \(y\) are not the same span). It is easy to verify that \(\text{overlap}_{d}\) indeed is expressible by a regular spanner.

The following dc, denoted \(d_{loc}\), states that the spans of locations are disjoint.

\[
d_{loc} := \text{overlap}_{d}[x, y] \rightarrow \neg (\operatorname{Loc}(x) \land \operatorname{Loc}(y))
\]

Similarly, the following dc, denoted \(d_{bp}\), states that spans of locations are disjoint from spans of persons.

\[
d_{bp} := \text{overlap}[x, y] \rightarrow \neg (\operatorname{Loc}(x) \land \operatorname{Per}(y))
\]

To specify a priority relation \(\succ\), we use what is called in [18] a priority generating dependency, or just \(pdg\) for short. Let \(S\) be a signature, and let \(R\) be a spanner representation system. A pdg in \(R\) (for \(S\)) has the form \(\forall x[P \rightarrow (\varphi(x) > \varphi'(x))]\), where \(x\) is a sequence of variables in SVars, \(P\) is a spanner specified in \(R\) with all of its variables in \(x\), and \(\varphi\) and \(\varphi'\) are atomic formulas over \(S\). Again, we usually omit the universal quantifier and write just \(P \rightarrow (\varphi(x) > \varphi'(x))\). And again, the semantics of \(P \rightarrow (\varphi(x) > \varphi'(x))\) is the obvious one: for all \((x, d)\) tuples \(\mu\), if \(P(d)\) contains the restriction of \(\mu\) to SVars\((P)\), then \(f \succ f'\) where \(f\) and \(f'\) are the facts that are obtained from \(\varphi(x)\) and \(\varphi'(x)\), respectively, by replacing every variable \(x\) with the span \(\mu(x)\).
Figure 3. The reader can verify that dcs
tionship between the person and the location).

In the relation Loc, spans that start
together, the longer one is preferred.

\[ p_{loc} := \rho[x, y] \rightarrow (\text{Loc}(x) \supset \text{Loc}(y)) \]

Here, \( \rho[x, y] \) is the following expression in REG.

\[ \pi_{x, y} \left( (\Sigma^* \cdot x \{ z \cdot \Sigma^* \} \cdot \Sigma^*) \triangleright \right.
\]
\[ \left. (\Sigma^* \cdot z \{ e \cdot \Sigma^+ \cdot y \{ \Sigma^* \} \cdot \Sigma^* \}) \vee \right. \]
\[ (\Sigma^* \cdot x \{ y(\Sigma^*) \Sigma^+ \} \cdot \Sigma^*) \]

Intuitively, the first disjunct says that \( x \) begins before \( y \),
because \( x \) begins with the empty span \( z \), and \( y \) begins
strictly after \( z \) begins. The second disjunct says that \( x \) and \( y \) begin together, but \( x \) ends strictly after \( y \) ends.

The following pgd, denoted \( p_{pgd} \), states that all the facts of
Loc are preferred to all the facts of Per (e.g., because the extraction
made for Loc is deemed more precise). We use the Boolean spanner
to true that is true on every
document.

\[ p_{pgd} := \text{true} \rightarrow (\text{Loc}(x) \supset \text{Per}(y)) \]

As we will discuss in Section 5.4, common resolution
strategies translate into a dc and a pgd, such that the
dc is binary, and the pgd defines priorities precisely on the
facts that are in conflict. To refer to such a case conveniently, we write
\( P \rightarrow (\varphi(x) \triangleright \varphi'(x)) \) to jointly
represent the dc \( P \rightarrow \neg(\varphi(x) \wedge \varphi'(x)) \) and the pgd
\( P \rightarrow (\varphi(x) \triangleright \varphi'(x)) \). We call such a constraint a
denial pgd.

**Example 5.4.** We use \( \text{contains}_{\rho}[x, y] \) to denote a
regex formula that produces all pairs \((x, y)\) of spans
where \( x \) strictly contains \( y \). Let \( \text{enclose}[x, y] \) denote a
specification in REG that produces all the triples \((z, x, y)\),
such that \( z \) begins where \( x \) begins and ends where \( y \)
ends. For presentation’s sake, we avoid the precise specification
of these formulas.

The following denial pgd, denoted \( d_{p_{enc}} \), states that
in the relation PerLoc, two facts are in conflict if the
span that covers the two elements (person and location) of
the first fact strictly contains that span that covers the
two elements of the second; in that case, the shorter span
is prioritized (since a shorter span indicates closer relationship
between the person and the location).

\[ \text{enclose}[z, x, y] \triangleright \text{enclose}[z', x', y'] \triangleright \text{contains}_{\rho}[z', z] \]
\[ \rightarrow \text{PerLoc}[x, y] \supset \text{PerLoc}[x', y'] \]

**Example 5.5.** Consider again the \( d \)-instance \( I \) of
Figure 3. The reader can verify that dcs \( d_{loc} \) and \( d_{pgd} \)
from Example 5.2, the pgds \( p_{loc} \) and \( p_{pgd} \) in Example 5.3
and the denial pgd \( d_{p_{enc}} \) of Example 5.4, together define
the conflicts and priorities discussed in Example 5.1
(Figure 5).

Let \( \mathcal{R} \) be a spanner representation system. An
extraction program in \( \mathcal{R} \), or just \( \mathcal{R} \)-program for short, is
similar to a basic \( \mathcal{R} \)-program, except that we now allow for
cleaning rules in addition to the Horn rules. A cleaning rule
has the form \( \text{CLEAN}(\delta_1, \ldots, \delta_k) \), where each
\( \delta_i \) is a dc or a pgd in \( \mathcal{R} \) (for convenience, we will also
allow denial pgds).

In the program of the following example, we specify
an extraction program \( \langle S, U, \varphi \rangle \) using only \( U \) (a sequence
of rules) along with a special \( \text{RETURN} \) statement that
specifies \( \varphi \). We then assume that \( S \) consists of precisely
the relation symbols that occur in the program.

**Example 5.6.** We now define the REG-program \( \mathcal{E} \)
of our running example. Intuitively, the goal of the program
is to extract pairs \((x, y)\), where \( x \) is a person and
\( y \) is a location associated with \( x \). The signature is, as
usual, that of Example 3.3. The sequence \( U \) of rules is
the following. Note that we are using the notation we
established in the previous examples.

1. \( \text{Loc}(x) \leftarrow \rho_{loc}[x] \) (Example 3.2)
2. \( \text{Per}(y) \leftarrow \rho_{pgd}[y] \) (Example 3.2)
3. \( \text{CLEAN}(d_{loc}, d_{pgd}, p_{loc}, p_{pgd}) \) (Examples 5.2 and 5.3)
4. \( \text{PerLoc}(x, y) \leftarrow \neg(\text{Per}(x) \wedge \text{Loc}(y) \wedge \text{precede}[x, y]) \)
   (Example 5.2)
5. \( \text{CLEAN}(d_{p_{enc}}) \) (Example 5.4)
6. \( \text{RETURN} \left(\text{PerLoc}(x, y)\right) \)

Note that lines 1, 2, and 4 are Horn rules, whereas lines 3
and 5 are cleaning rules.

Let \( \mathcal{E} = \langle S, U, \varphi \rangle \) be an \( \mathcal{R} \)-program and let \( d \) be a
document. Suppose that \( U = \{u_1, \ldots, u_m\} \). Let \( I_0 \) be the
singleton \( \{I_0\} \), where \( I_0 \) is the empty instance over
\( S \). For \( i = 1, \ldots, m \), we denote by \( I_i \) the result of
executing the rules \( u_1, \ldots, u_i \) as we describe below. Since
the cleaning operation can result in multiple instances
(optional repairs), each \( I_i \) is a set of \( d \)-instances, rather
than a single one. For \( i > 0 \) we define the following.

1. If \( u_i \) is the Horn rule \( R(x_1, \ldots, x_n) \leftarrow \alpha_1 \wedge \cdots \wedge \alpha_k \),
   then \( I_i \) is obtained from \( I_{i-1} \) by adding to each
   \( I \in I_{i-1} \) all the facts (over \( R \)) that are obtained by
evaluating the rule over \( I \).
2. If \( u_i \) is the cleaning rule \( \text{CLEAN}(\delta_1, \ldots, \delta_k) \), then
   \( I_i \) is obtained from \( I_{i-1} \) by replacing each \( I \in I_{i-1} \) with all the
   optimal repairs of \( I \), as defined by the conflict hypergraph and priorities implied
   by all the \( \delta_j \).

---

\(^3\)Again, our program is simplistic, for the sake of presentation.
Recall that a spanner is a function that maps a document into a \((V, d)\)-relation (see Section 2). An extraction program acts similarly, except that a document is mapped into a set of \((V, d)\)-relations (since it branches into multiple optimal repairs); these are all the possible resulting relations \(\varphi\). For a more precise definition of the output of an extraction program, see [18]. In practice, the common case is where the extraction program produces precisely one \((V, d)\)-relation, and then we will view the extraction program simply as a spanner.

**Example 5.7.** Consider again the REG-program \(E\) of Example 5.6. We will now follow the steps of evaluating the program \(E\) on the document \(d\) of our running example (Figure 1). It turns out that, in this example, each \(I_i\) is a singleton, since every cleaning operation results in a unique optimal repair. Hence, we will treat the \(I_i\) as instances.

1. In \(I_1\), the relation \(Loc\) is as shown in Figure 3, and the other two relations are empty.
2. In \(I_2\), the relations \(Loc\) and \(Per\) are as shown in Figure 3, and \(PerLoc\) is empty.
3. In \(I_3\), the relations \(Loc\) and \(Per\) are as shown in Figure 6, and \(PerLoc\) is empty. The cleaning process is described throughout Sections 5.1 and 5.2.
4. In \(I_4\), the relations \(Loc\) and \(Per\) are as in \(I_3\), and \(PerLoc\) is as shown in Figure 3.
5. \(I_5\) is the instance shown in Figure 6.

The result \(\mathcal{E}(d)\) is the \(((x, y), d)\)-relation that has two mappings: the first maps \((x, y)\) to \(([1, 7], [13, 28])\), and the second to \(([30, 40], [46, 48])\).

### 5.3 Cleaning in REG-Programs

We now discuss fundamental properties of extraction programs, where we focus on the class of REG-programs.

In the framework of prioritized repairs [30], the priority relation is assumed to be acyclic. We did not make such an assumption, and a pgd can indeed define a cyclic priority relation in a given program. We would like to be able to test whether acyclicity is guaranteed, but unfortunately, as the next theorem implies, no such algorithm exists for general pgds.

Let \(c\) be a cleaning rule. We say that \(c\) is acyclic if, for every document \(d\) and \(d\)-instance \(I\) over \(S\), the priority relation implied by the pgds of \(c\) is acyclic.

**Theorem 5.8.** [18] Whether a pgd in REG is acyclic is co-recursively enumerable but not recursively enumerable. In particular, it is undecidable.

Recall that a spanner maps a document \(d\) into a \((V, d)\)-relation, for a set \(V\) of variables, while an extraction program maps \(d\) into a set of \((V, d)\)-relations. The next property we discuss for extraction programs is that of unambiguity, which is the property of having a single possible world when the program is evaluated over any given document. Formally, we say that extraction program \(E\) is unambiguous if \(\mathcal{E}(d)\) is a singleton \((V, d)\)-relation for every document \(d\). We may view an unambiguous extraction program \(E\) simply as a specification of a spanner. The following theorem states that, unfortunately, in the presence of cleaning rules unambiguity cannot be verified for regular extraction programs.

**Theorem 5.9.** [18] Whether a REG-program is unambiguous is co-recursively enumerable but not recursively enumerable. In particular, it is undecidable.

We now give a sufficient and decidable condition for unambiguity, in the case where acyclicity is guaranteed. Let \(I\) be a \(d\)-instance over a signature \(S\). Let \(H\) and \(>\) be a conflict hypergraph for \(I\) and a priority relation over \(I\), respectively. We say that \((>, H)\) satisfies the minimum property if every hyperedge \(h\) of \(H\) contains a minimum element, that is, an element \(a\) such that \(b > a\) for every member of \(b\) other than \(a\). Let \(c\) be a cleaning rule. We say that \(c\) is minimum generating if, for every document \(d\) and \(d\)-instance \(I\) over \(S\), for the priority relation \(>\) and conflict hypergraph \(H\) implied by \(c\) we have that \((>, H)\) satisfies the minimum property.

We note that for acyclic priority relations, the minimum property is less strict than the totality property that Sta-worko et al. [30] gave as a condition for unambiguity. We have the following theorem.

**Theorem 5.10.** [18] Let \(E\) be an \(R\)-program for some spanner representation system \(R\). If every cleaning rule of \(E\) is acyclic and minimum generating, then \(E\) is unambiguous.

In addition, we have shown that the property of being minimum generating is decidable for regular spanners.

**Theorem 5.11.** [18] Whether a given cleaning rule in REG is minimum generating is decidable.

Unfortunately, the property of being acyclic is undecidable, as stated in Theorem 5.8, and so is the property of being both acyclic and minimum generating. Hence, as future research it is of interest to find decidable properties that imply these two properties.

Next, we address the question of whether cleaning rules increase the expressive power of extraction programs. Let \(R\) be a spanner representation system. A cleaning rule \(c\) defined in \(R\) is said to be \(R\)-disposable if the following holds: for every \(R\)-program \(E\) that has \(c\) as its single cleaning rule, there exists a basic (non-cleaning) \(R\)-program that is equivalent to \(E\). Of course, if every cleaning rule of \(E\) is \(R\)-disposable, then \(E\) is equivalent to a basic \(R\)-program.

We say that a denial pgd \(p\) is \(R\)-disposable if the cleaning rule that consists of only \(p\) is \(R\)-disposable.
The following theorem implies that cleaning rules, and in fact a single acyclic denial pgd, increase the expressive power of regular extraction programs. Recall that a program that uses an acyclic denial pgd as its single cleaning rule is unambiguous (Theorem 5.10).

**Theorem 5.12.** [18] There exists an acyclic denial pgd in REG that is not REG-disposable.

### 5.4 JAPE Controls

JAPE [14] is an instantiation of the Common Pattern Specification Language (CPSL) [4], a rule based framework for IE. A JAPE program (or “phase”) can be viewed as an extraction program where all the relation symbols are unary. JAPE has several built-in cleaning strategies called “controls.” Here, we will define these strategies in our own terminology—denial pdgs.

JAPE provides four controls (in addition to the All control stating that no cleaning is to be applied). These translate to the following denial pdgs. Here, $R$ is assumed to be a unary relation in an extraction program. Under the *Appelt* control, $R(x) \triangleright R(y)$ holds if (1) $x$ and $y$ overlap and $x$ starts earlier than $y$, or (2) $x$ and $y$ start at the same position but $x$ is longer than $y$. The same strategy is used is also provided by SystemT [10] (as a “consolidator”). The first control is similar to *Appelt* with “longer” replaced with “shorter.” The *Brill* control is similar to *Appelt*, with the exclusion of option (2); that is, $R(x) \triangleright R(y)$ holds if $x$ and $y$ overlap and $x$ starts earlier than $y$. The *Once* control states that a single fact should remain in $R$ (unless $R$ is empty), which is the one that starts earliest, where a tie is broken by taking the one that ends earliest. Hence, $R(x) \triangleright R(y)$ if and only if $x$ is that remaining fact and $x \neq y$.

It is easy to show that each of the above denial pdgs is acyclic, and can be expressed in REG. For example, the *Appelt* control is presented in Example 5.3 with $R$ being the relation symbol Loc. While the JAPE controls can significantly simplify the programming of spanners, they do not add expressive power to regular programs, as the following theorem states.

**Theorem 5.13.** [18] Each of the denial pdgs that correspond to the four JAPE controls is REG-disposable.

### 5.5 Regular Spanners and POSIX

A regex formula $\gamma$ defines a spanner by considering all possible ways that input document $d$ can be matched by $\gamma$; that is, it considers all possible (functional) parse trees of $\gamma$ on $d$. Each such parse tree generates a new $(V, d)$-tuple, where $V = SVars(\gamma)$, in the resulting span relation. In contrast, regular-expression pattern-matching facilities of common UNIX tools, such as *sed* and *awk*, or programming languages such as Perl, Python, and Java, do not construct all possible parse trees. Instead, they employ a *disambiguation policy* to construct only a single parse tree among the possible ones. As a result, a regex formula in these tools always yields a single $(V, d)$-tuple per matched input document $d$ instead of multiple such tuples.\(^4\)

In this section, we discuss the POSIX disambiguation policy [20, 23], a policy which is followed by all POSIX compliant tools such as *sed* and *awk*. Formalizations of this policy have been proposed by Vansummeren [31] and Okui and Suzuki [26], and multiple efficient algorithms for implementing the policy are known [12, 24, 26].

POSIX disambiguates as follows when matching a document $d$ against regex formula $\gamma$.\(^5\) A formal definition may be found in [26, 31]. If $\gamma$ is one of $\emptyset$, $\epsilon$, or $\sigma \in \Sigma$ then at most one parse tree exists; disambiguation is hence not necessary. If $\gamma$ is a disjunction $\gamma_1 \lor \gamma_2$, then POSIX first tries to match $\gamma$ against $\gamma_1$ (recursively, using the POSIX disambiguation policy to construct a unique parse tree for this match). Only if this fails it tries to match against $\gamma_2$ (again, recursively). If, on the other hand, $\gamma$ is a concatenation $\gamma_1 \cdot \gamma_2$ then POSIX first determines the longest prefix $d_1$ of $d$ that can be matched by $\gamma_1$ such that the corresponding suffix $d_2$ of $d$ can be matched by $\gamma_2$. Then, $d_1$ (respectively, $d_2$) is recursively matched under the POSIX disambiguation policy by $\gamma_1$ (respectively, $\gamma_2$) to construct a unique parse tree for $\gamma$. When $\gamma$ is a Kleene closure $\delta^*$, there are two cases. If $d$ is empty, then the entire pattern $\gamma$ is taken to match $d$ (irrespective of whether $\delta$ itself matches $d$), and disambiguation is not necessary. If, on the other hand, $d$ is nonempty, then POSIX expands $\gamma$ to $\delta \cdot \delta^*$. In line with the rule for concatenation, it hence first determines the longest prefix $d_1$ of $d$ that can be matched by $\delta$ such that the corresponding suffix $d_2$ of $d$ can be matched by $\delta^*$. Then, a unique parse tree for $d$ against $\gamma$ is constructed by matching $d_1$ recursively against $\delta$ and $d_2$ against $\delta^*$.

The following example illustrates the POSIX policy.

**Example 5.14.** Consider $\gamma = x\{0 \lor 01\} \cdot y\{1 \lor \epsilon\}$ and $d = 01$. Under the POSIX disambiguation policy, subexpression $x\{0 \lor 01\}$ will match as much of $d$ as possible while still allowing the rest of the expression, namely $y\{1 \lor \epsilon\}$, to match the remainder of $d$. As such, $x\{0 \lor 01\}$ will match $d$ entirely, and $y\{1 \lor \epsilon\}$ will match the empty string. We hence bind $x$ to the span $[1, 3]$ and $y$ to $[3, 3]$.

\(^4\)While our syntax $x[\gamma]$ for variable binding is not directly supported in these tools, it can be mimicked through the use of so-called parenthesized expressions and submatch addressing.

\(^5\)For simplicity, we restrict ourselves here to the setting where the entire input is required to match $\gamma$. Our results naturally extend to the setting where partial matches of $d$ against $\gamma$ are sought.
As another example, when \( \gamma = (x\{0\} \cdot y\{(1 \lor \epsilon)\}) \lor (x\{01\} \cdot y\{(1 \lor \epsilon)\}) \) and \( d = 01 \), we bind \( x \) to the span \([1, 2]\) and \( y \) to the span \([2, 3]\) under the POSIX disambiguation policy. \( \square \)

By \( \text{posix}[\gamma] \) we denote the spanner represented by the regex formula \( \gamma \) under the POSIX disambiguation policy; this is the spanner such that \( \text{posix}[\gamma](d) \) is empty if \( d \) cannot be matched by \( \gamma \), and consists of the unique \((V, d)\)-tuple resulting from matching \( d \) against \( \gamma \) under the POSIX disambiguation policy otherwise.

The following theorem shows that the POSIX policy can be expressed in our cleaning framework.

**Theorem 5.15.** \([18]\) For all regex formulas \( \gamma \) there exists a REG-program \( E \) such that for every document \( d \),

\[
E(d) = \{ \text{posix}[\gamma](d) \}.
\]

While proving Theorem 5.15, we have observed that every cleaning rule we used in \( E \) is REG-disposable. Moreover, since the spanner \( \text{posix}[\gamma] \) is hierarchical, it follows by Theorem 4.2 that \( \text{posix}[\gamma] \) is itself definable in RGX by a regex formula \( \delta \). We then conclude the following theorem about POSIX, which is of interest independently of our framework.

**Theorem 5.16.** \([18]\) For every regex formula \( \gamma \), the spanner \( \text{posix}[\gamma] \) is definable in RGX.

### 6. STRING EQUALITY

In this section we discuss the enrichment of regular spanners with the binary string-selection operator, denoted \( \preceq \). Given a spanner \( P \) and two variables \( x, y \in \text{SVars}(P) \), the application of \( \preceq_{x,y} \) selects all the tuples \( \mu \) in which \( d_\mu(x) = d_\mu(y) \). A **core spanner** \([19]\) is a spanner definable in the algebra \( \text{RGX}(\Sigma, \pi, \mu, \preceq) \). We denote this algebra by \( \text{Core} \).

It follows immediately from known literature on finite-state automata that core spanners have a strictly greater expressive power than regular spanners; that is, every regular spanner is a core spanner, and there are core spanners that are not regular \([19]\). An example of a non-regular core spanner is the following spanner, extracting all the pairs of spans with equal strings:

\[
\preceq_{x,y} \left( (\Sigma^* x \{ \Sigma^* \}) \Sigma^* \right) \times (\Sigma^* y \{ \Sigma^* \}) \Sigma^* \}
\]

Recall from Theorem 4.4 that regular spanners are closed under difference. The following theorem states that this is no longer the case for core spanners.

**Theorem 6.1.** \([19]\) Assume that the alphabet \( \Sigma \) contains at least two symbols. Core spanners are not closed under difference; that is,

\[
[\text{RGX}(\Sigma, \pi, \mu, \preceq)] \not\subseteq [\text{RGX}(\Sigma, \pi, \mu, \preceq_{x,y})].
\]

Next, we discuss the proof of Theorem 6.1. As noted in \([19]\), the authors originally believed that the way to prove Theorem 6.1 would be to show that core spanners cannot simulate string inequality (i.e., select the tuples \( \mu \) in which \( d_\mu(x) \neq d_\mu(y) \)). However, surprisingly, it turned out that this argument is false.

**Proposition 6.2.** \([19]\) Core spanners are closed under the string-inequality operator.

As a part of the proof of Theorem 6.1, we established the following lemma, which is of independent interest.

**Lemma 6.3.** Every core spanner is definable by an expression of the form \( \pi_Y SP \), where \( P \) defines a regular spanner, \( V \subseteq \text{SVars}(P) \), and \( S \) is a sequence of selections \( \preceq_x, y \in \text{SVars}(P) \).

The proof of Theorem 6.1 then completes as follows. An easy observation is that core spanners are closed under the substring-of selection operator (i.e., select the tuples \( \mu \) in which \( d_\mu(x) \) is a substring of \( d_\mu(y) \)). But using Lemma 6.3 we have proved the following theorem.

**Theorem 6.4.** \([19]\) Assume that the alphabet \( \Sigma \) contains at least two symbols. Core spanners are not closed under the not-a-substring-of binary operator.

We complete this section with the following theorem, which implies that disposability of the JAPE controls no longer holds in the case of core spanner.

**Theorem 6.5.** \([18]\) None of the JAPE denial pgds is Core-disposable.

### 7. CONCLUDING REMARKS

We conclude this paper by some observations relating spanners to other formalisms, and some open questions.

Many practical regular expression pattern matching engines (such as the ones found in \text{sed}, \text{awk}, \text{Perl}, \text{Java} and \text{Python}) support a feature called backreferences. Using this feature, variables can not only bind to a substring during matching, but can also be used to repeat a previously matched substring. Regular expressions that have this feature are called *extended regular expressions* (\text{xregex} for short) \([1, 8, 9]\). It is known that \text{xregex} can recognize non-regular languages, such as \( \{ ss \mid s \in \Sigma^* \} \). Note that this language can be recognized by a Boolean core spanner. In \([19]\) we established that \text{xregex} can also recognize languages that are not recognizable by any Boolean core spanner. The reverse question, whether Boolean core spanners can recognize languages that are not recognizable by \text{xregex}, is still open.

Various languages for querying semi-structured and graph databases are based on regular expressions. A simple form of such queries are the *regular path queries*...
(RPQs) that are applied to directed graphs with labeled edges [11, 13]. An RPQ identifies node pairs connected by a path such that the word formed by the edge labels belongs to a specified regular language. A conjunctive regular path query (CRPQ) applies conjunction and existential quantification (over nodes) to RPQs [11], and has been the subject of much investigation.

Superficially speaking, spanners and CRPQs are inherently different concepts: spanners operate on strings while CRPQs operate on graphs (directed, edge-labeled graphs); and the variables in the spanner world represent spans, while those in the CRPQ world represent nodes. However, it is possible to adjust CRPQs to represent spanners [19]. In terms of the data model, a string can be viewed as a special case of a graph, namely a simple path. Formally, given a string $s = \sigma_1 \cdots \sigma_n$, we denote by $p(d)$ the simple path $1 \rightarrow 2 \rightarrow \cdots \rightarrow n + 1$ (with the natural numbers $1, \ldots, n + 1$ as nodes), where for $i = 1, \ldots, n$ the label of the edge $i \rightarrow i + 1$ is $\sigma_i$. For technical reasons, it is necessary to mark the begin node 1 and end node $n$ in this simple path with the two loops $1 \rightarrow 1$ and $n + 1 \rightarrow (n + 1)$, labeled with new labels $\triangleright$ and $\triangleleft$ (not in the alphabet $\Sigma$). On this so-called marked path, a CRPQ can define a spanner over a set of span variables $V$ by introducing, for each $x \in V$, two CRPQ variables: one that will indicate the start position of the span matched by $x$ and one that indicates the end position of the span matched by $x$. Using this representation of spanners through CRPQs, one can show that $\langle \text{REG} \rangle$ is exactly captured by unions of CRPQs, while $\langle \text{Core} \rangle$ is exactly captured by unions of CRPQs extended with string equality [19].

As we have seen in this article, we have identified several representation systems for regular and core spanners that are equivalent in expressive power. While our proofs of these equivalences describe effective translations between the representation systems, it would be interesting to study the inherent complexity of these translations in order to establish their relative succinctness. A second question that deserves further attention is the complexity of evaluating spanners expressed in the various representation systems.

8. REFERENCES