Spatial Operators*

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Abstract

This paper discusses issues related to the integration of spatial operators into the new generation of SQL-like query languages. Starting from spatial data models, current spatial extensions of query languages are briefly reviewed and research directions are highlighted. A taxonomy of requirements to be satisfied by spatial operators is proposed with emphasis on users' needs and on the introduction of data uncertainty support. Further, spatial operators are classified into the three important categories of topological, projective, and metric operators and for each of them the state of the art is outlined.

1 Introduction

In spatial applications, the data of interest concerns the geometry of objects embedded in space. The scale in which the objects are represented determines the type of the application: small scale pertains to CAD/CAM applications, while large scale pertains to GIS applications. After decades of various independent approaches in this area, the software community is now recognizing the importance of basing spatial applications on database technology. Therefore, the subfield of *spatial databases* is becoming extremely challenging, as is testified by state-of-the-art papers [Güt94, S+99].

Issues connected with the definition of *spatial Query Languages* (QLs) are among the relevant research topics with motivations ranging from the need of providing the user with an interactive tool for data retrieval which is independent of the physical organization of data, to the enhancement of interoperability across different systems between nodes of a world-wide distributed network. Recent proposals to extend the SQL92 language with spatial operators aim at integrating capabilities to handle spatial data directly in the upcoming SQL3 standard. The most noticeable of such proposals is that by the OpenGIS Consortium [The98].

The interest of the database community in extending standard QLs with spatial operators goes back to the early 80's. Since then, many contributions have been made, coming mainly from the field of pictorial databases. The PICQUERY+ language and the Knowledge-Based Spatial and Temporal query language are two excellent examples of such a trend [C+93, CHCT98].

The problem common to all these contributions is that they have not been preceded by a specific investigation of the requirements that spatial operators should satisfy (e.g., expressiveness and consistency, see Section 3). The work by Egenhofer [Ege94] is a relevant example of a contribution towards a spatial extension of standard SQL that comes from the field of GISs. That paper is the first to have introduced a set of requirements for spatial operators, basically concerning graphical manipulation and visualization of query input/output.

Spatial operators are used to capture all the relevant geometric properties of objects embedded in the physical space and the relations between them, as well as to perform spatial analysis. Operators that apply to all primitive geometric data types (namely, points, lines, and regions) can be defined at a more general level, while other operators are specific to derived data types (e.g. networks). The more basic type of operators are set-oriented operators (intersection, union, difference) that were also present in early proposals of spatial query languages.

Although the OpenGIS extension of SQL introduces several spatial operators that are extracted

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from previous research on spatial relations (e.g., [CDF96b]), more theoretical research is needed to define a complete set of operators. In this paper, we concentrate on spatial operators that can be defined at a general level for all elementary geometric data types (points, lines, regions) and are used to assess their geometric properties. More elaborate spatial operators can be defined based on the elementary types, that can apply to aggregate data types (e.g. networks) and can be used to perform more complex kinds of spatial data analysis.

We can distinguish between *unary* and *binary* spatial operators, which are applied, respectively, to assess properties of single objects or relations between objects. Overall, they can be structured along the three orthogonal "dimensions" proposed by Clementini and Di Felice [CDF97b] and, accordingly, hereafter called:

- *topological* operators: through topology we can express predicates about the connection, the number of components, the presence/absence of holes as well as topological relations (which describe whether two objects intersect or not, and, in the former case, how they intersect);

- *projective* operators: through projective operators, we can express predicates about the concavity/convexity of objects as well as other spatial relations (e.g., being inside the concavity of a given object);

- *metric* operators: through metrics, we can express predicates about the compactness or symmetry properties of objects as well as *distance* and *directional* relations.

The paper is organized as follows. We start (Section 2) with a discussion about models of the physical space and, in particular, of the OpenGIS proposal which represents an effort of achieving in the GIS community a wider degree of standardization in the modeling of the geometry of objects. In the same section, we point out that a new spatial data model should be introduced to deal with uncertainty in data. Then a list of requirements that spatial operators should satisfy is given (Section 3). Subsequently, the current knowledge about spatial operators is briefly recalled for the three categories of geometric properties previously mentioned (i.e., topological, projective and metric) and future research directions are highlighted for each category (Sections 4–6).

2 Spatial data models

There are two common models of the physical space: *field*-oriented and *object*-oriented. From a database perspective, object-oriented models are definitely the best choice. Object-oriented models treat the physical space as it is populated by discrete, identifiable, spatially-referenced entities. The geometry is the fundamental peculiarity of spatial data. For integrating geometric aspects into a data model it is necessary to represent spatial objects (in the sense of the application) as "objects" (in the sense of the DBMS) having at least one attribute of a "geometric" type. In practical terms, this means that the data model must support, besides common data types (e.g., `integer`, `float`, `string`, etc.), *geometric* data types as well. The OpenGIS proposal, in particular, refers to the following geometric data types: `Point`, `LineString`, `Polygon`, `MultiPoint`, `MultiLineString`, `MultiPolygon`, etc. (Figure 1 shows the full hierarchy of geometric data types).

Table 1 shows the operators that are defined inside the class Geometry. Their adoption allows the formulation of queries which mix both spatial and non-spatial predicates. For example, let us suppose we are interested in knowing all the parcels which satisfy a "containment" topological relation with a fixed soil unit category (let say "4"). The database can be queried through the following SQL query:

```
SELECT Parcel.name
FROM Parcel, SoilUnit
WHERE Within(Parcel.geo, SoilUnit.geo)
AND SoilUnit.category=4;
```

Up to this point, spatial data models have assumed that the extent, and hence the boundary, of spatial objects is precisely determined. This leads to database objects which have exact geometry (commonly called: objects with a *crisp boundary* or simply crisp objects). Unfortunately, this is a simplification of the reality that is not acceptable in many cases. A useful categorization of uncertainty in spatial data comes from [Wor98]: incompleteness, inconsistency, vagueness, imprecision, and error.

A new geometric model is needed that overcomes the limits of the current models of spatial databases, which traditionally are a collection of lines (points, polylines and polygons). Also the OpenGIS specification for SQL mentioned above, is based on a geometric model supporting objects with a crisp boundary.

The proposal of many recent research papers is to introduce *broad boundaries* replacing crisp ones [Sch96b, ES97, CDF96a, CG96]. Broad boundaries

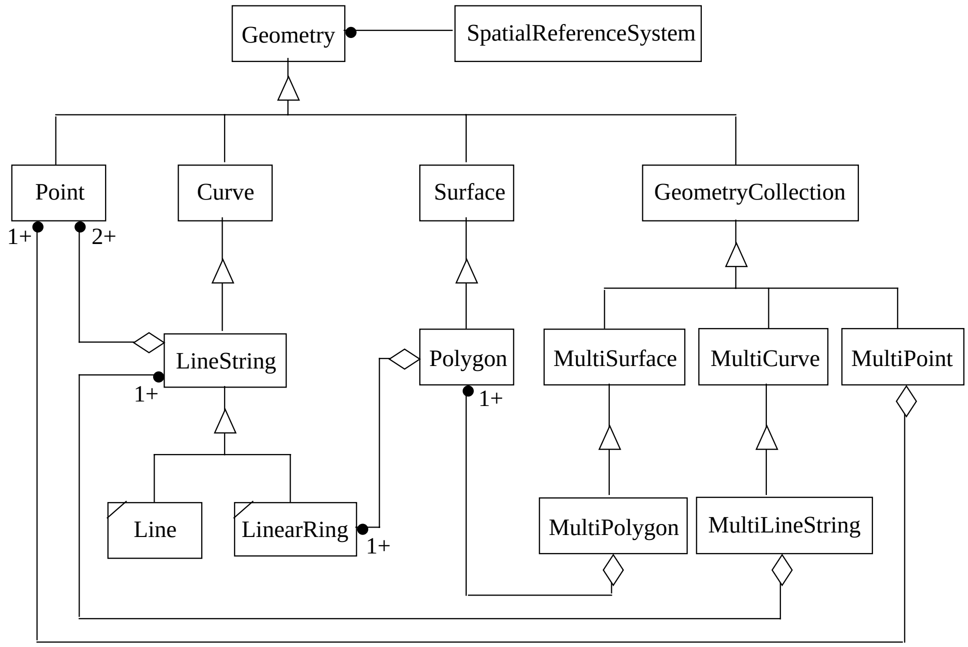

Figure 1: A geometric data type hierarchy, [The98]

Table 1: Operators defined on the class Geometry by the OpenGIS, [The98].

Basic operators	SpatialReference	Returns the Reference systems of the geometry
	Envelope	The minimum bounding rectangle of the geometry
	Export	Convert the geometry into a different representation
	IsEmpty	Tests if the geometry is the empty set or not
	IsSimple	Returns True if the geometry is simple
	Boundary	Returns the boundary of the geometry
Topological operators	Equal	Tests if the geometries are spatially equal
	Disjoint	Tests if the geometries are disjoint
	Intersect	Tests if the geometries intersect
	Touch	Tests if the geometries touch each other
	Cross	Tests if the geometries cross each other
	Within	Tests if the given geometry is within another given geometry
	Contains	Tests if the given geometry contains another given geometry
	Overlap	Tests if the given geometry overlaps another given geometry
	Relate	Returs True if the spatial relationship specified by the 9-Intersection matrix holds.
Spatial analysis operators	Distance	Returns the shortest distance between any two points of two given geometries
	Buffer	Returns a geometry that represents all points whose distance from the given geometry is less than or equal to the specified distance
	ConvexHull	Returns the convex hull of the given geometry
	Intersection	Returns the intersection of two given geometries
	Union	Returns the union of two given geometries
	Difference	Returns the difference of two given geometries
	SymDifference	Returns the symmetric difference of two given geometries

absorb all the uncertainty commonly present in spatial data and allow computations without rough simplifications of the reality. The advantage of this approach is that it can be implemented on existing database systems at a reasonable cost: the new model can be seen as an extension of the existing geometric models.

In this paper, we refer to a model of this kind. For example, to extend the OpenGIS specifications in the direction of taking into account uncertainty, first of all new classes need to be added to the class hierarchy of Figure 1. Figure 2 proposes a possible extension made in terms of the classes `BBPolygon` and `BBMultiPolygon`, respectively, which have multiple inheritance from their crisp counterparts and from the class `BBGeometry` (the latter being a subclass of `Geometry`).

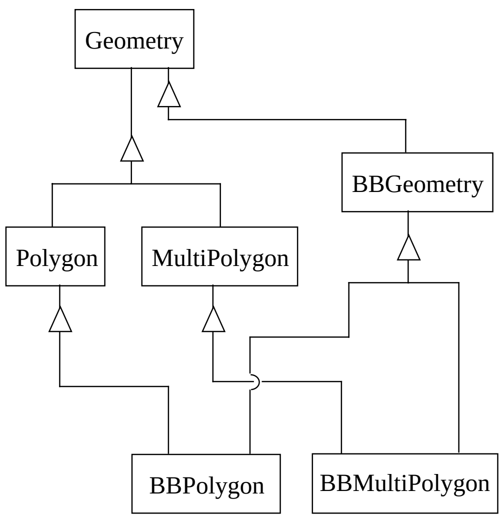

Figure 2: An extension of the geometric data type hierarchy of Figure 1 towards broad boundaries.

3 Requirements for spatial operators

Spatial operators to be implemented in a QL should satisfy several requirements. The current state of the art prevents us from the enucleation of a set of requirements that can be thought of as exhaustive of the different aspects that are involved. Nevertheless, from the analysis of the existing literature in the fields of spatial databases, multimedia databases, and GISs, we can emphasize the importance of nine basic requirements that spatial operators should fullfill. Such requirements, globally, meet users' expectations in terms of simplicity of usage, expressiveness and linguistic and cognitive soundness, as well as the simplicity of implementation of the operators into a QL. We give priority to the possibility of extracting (categories of) objects from the database according to specific geometric constraints (such as number of holes/components) and/or spatial relations with other objects, taking into account the characteristics of real data sets, such as their intrinsic level of uncertainty. In detail, the nine requirements are the following:

- *small set of operators*: the operators should be small in number in order to reduce the learning time of users. In connection with topological operators, for example, the proposals available in the literature which enumerate all the realizable geometric configurations between pairs of objects are not immediately applicable for the definition of operators to be embedded into an actual QL. For instance, the 9-intersection method proposed by Egenhofer [EH91] distinguishes among 56 cases between pairs of 2-D geometric objects (namely, points, lines, and areas). Obviously, for humans 56 operators would be too much to be used in a reasonable manner in a QL;

- *expressiveness*: the operators should enable the user to formulate a significant range of queries. Generally spatial queries are more complex than non-spatial ones and this because of the geometry. Studies about the expressiveness of spatial QLs are of primary importance. They should be guided by formal criteria defining the domain of geometric configurations that are identifiable by combining the available operators. [CDF95] is a pioneeristic paper which investigates the expressiveness of a certain number of methods for representing topological operators all based on point-set topology;

- *consistency*: the operators should not give rise to ambiguous computations nor to inconsistent results. This is essentially achieved by relying on formal approaches for spatial relations that ensure theoretical properties such as completeness and mutual exclusiveness of relationships. For example, in the case of topological relationships, point-set topology is the common formal basis of existing proposals (e.g., [CDF96b, EF91, CDFC95]);

- *generality*: the operators should be defined at the level of abstract geometric data types and, hence, be application-independent. Much work has been done in the last decade on the formalization of spatial data models which support geometric data types, such as point, lines, and regions [GS95, CDF96b]. In these data models, real objects can be seen as instances of geometric data types. Spatial operators defined in such a framework are general enough to be application-independent;

- *hierarchical structuring*: the QL should provide the user with a hierarchical set of operators making it possible to pose queries at various levels of granularity in which geometric details vary from less detailed to more detailed. This allows users to follow a process of progressive refinement of query results. High level operators can be used for a fast screening of the data and more detailed operators to restrict the answer. Several models have been proposed, especially with reference to topological and distance relations [CDFK00, CDFH97], that can be used for defining hierarchically structured operators;

- *imprecise matching*: the QL should explicitly contain operators handling the vagueness in query formulation. This might be accomplished by measuring the similarity of spatial configurations with respect to the criteria specified by the user in the query. Similarity criteria may involve geometric aspects of the objects (such as shape or topological properties). Issues about imprecise matching are especially studied in the application context of multimedia databases: in [AMS98], for example, the authors introduced, among others, an operator (`sim`) that is able to measure the similarity between the object specified in the query and the instances stored in the database returning a value in the interval [0,1];

- *linguistic and cognitive soundness*: the operators should conform to widely accepted linguistic use of spatial terms and to a cognitive basis for spatial concepts in order to enhance the ease of use. Knowledge about parameters that play a significant role in the selection of spatial predicates by people in describing spatial relations is relevant in order to develop suitable models and to calibrate them to fit human intuition. *Naive Geography* is the field of study that is concerned with formal models of the common-sense geographic world [EM95]. In [ES98], Egenhofer and Shariff give a tangible contribution towards the formalization of people's usage of spatial predicates in natural language;

- *qualitativeness*: the operators should enable users to formulate queries dealing with the qualitative aspects of spatial objects and relations, complementing in such a way the more ordinary quantitative aspects. Up to now, existing QLs are useful in answering "exact" metric-based queries, but they cannot handle the way people communicate in everyday life. Indeed, queries like: "Display all relevant tourist spots located in Tuscany and *close* to Florence (i.e., reachable quickly by car)" are not allowed. Recently, contributions dealing with qualitative spatial relations have appeared mainly in connection with orientation and distance relations (e.g., [PS94, CDFH97, GE00]);

- *support of uncertainty*: besides operators that apply to crisp objects, also operators able to deal with objects with a broad boundary are of great interest in future spatial QLs. To limit users' mental overhead, it is highly recommended to keep as small as possible the number of ad hoc operators valid only for objects with a broad boundary (e.g., `innerArea`, `broadBoundaryWidth`), and to overload the operators valid for crisp objects as well (e.g., `intersect`, `distance`, `north_of`, `area`).

4 Topological operators

Topological properties are those that are invariant to topological transformations (1-to-1 bicontinuous functions), i.e., those properties which do not change after transformations like rotation, translation, scaling, and rubber sheeting. Topology can be considered the most primitive kind of spatial information, since a change in topology implies a change in other geometric aspects, while the opposite is not true. For such a reason, topological relations have been studied extensively during the last decade.

The theoretical studies on topological relations constitute the basis for defining the topological operators to be included in a spatial QL. The passage from relations to operators is not always obvious. In fact, the initial proposals to represent topological relations (e.g., the 4-intersection and the 9-intersection models [EF91, EH91]), although satisfying the consistency requirement, were not meeting other requirements on behalf of the user, such as giving rise to a small set of relations. The CBM proposal [CDF96b] was designed to make available to users a small set of operators (five binary operators - `touch`, `in`, `cross`, `overlap`, and `disjoint` - and three unary operators for boundaries), while maintaining the full expressive power, and, for this reason, it has been integrated in the OpenGIS extension of SQL. In [CDFC95], the CBM was extended with other two operators for handling separate components of spatial objects.

Topological relations between objects with a broad boundary and the corresponding topological operators are still a subject for research. The first results appeared in [CDF97a]. In such a paper, the 9-intersection model has been extended to simple regions with a broad boundary, totaling 44 different relations. Topological operators can be hierarchically structured in several levels, where the base level offers operators able to check for detailed topological relations between regions with a broad boundary using the extended 9-intersection model, and the higher levels offer more abstract operators that allow users to query uncertain spatial data independently of the underlying geometric data model. Very recently, Clementini *et al.* [CDFK00] proposed a three-level hierarchy of topological operators where the intermediate level is obtained by a specific clustering of the base relations and the top level is made up of the CBM operators (which are still valid for objects with a broad boundary).

In a recent paper [CDF98], the authors established a conceptual framework of reference for topological properties by proposing a set of seven *topological invariants* which is proven to fully characterize the binary topological relations among objects embedded in the plane. Topological invariants consent hierarchical structuring: there are general invariants (such as the *intersection content*) that allow a coarse classification of topological configurations and more detailed invariants (such as the *dimension* of a two line intersection component) that allow finer topological distinctions. Further research is needed to understand which other spatial operators can be extracted from the topological invariants mentioned in [CDF98] besides those already known.

5 Projective operators

Projective operators are the less studied and supported in current proposals on spatial QLs, but probably their importance is underestimated. In fact, projective properties, on which projective operators should be based, cover a wide range of geometric aspects that cannot be expressed in pure topological terms, but, being more primitive than metric ones, can still be recognized without the power of a metric space. The basic projective invariant is for any three points to maintain their order in the plane (collinearity is a special case). As a consequence, projective properties are related to being straight or curvilinear, to the number of vertices of a polygon, to the number of concavities of a contour. A basic projective operator is the *convex hull* of a region, which can be used also for establishing the relative positions of objects with respect to their concavities.

Cohn and others [CRCB93] define a wide set of spatial relations by using the convex hull primitive that are more detailed than topological relations based on connectedness alone. In this manner they can distinguish among regions that are inside, partly inside, or outside another region's convex hull whilst not overlapping at all with the other region.

The following SQL-like query (asking about houses whose geometry is inside that of a given forest) is a representative sample of usage of a projective operator concerning the convex hull:

```
SELECT House.id
FROM Forest, House
WHERE insideConvexHull (House.geo,
      Forest.geo);
```

Also the approach in [Sch96a] makes use of projective properties for qualitative description of geometric objects: the primitive being used is the distinction between the left and right side of a line in the plane.

6 Metric operators

This category of spatial operators provides a more specific description of the objects' geometry than the previous two. Metric operators can be used to measure some global properties of single objects (such as the area, the relative size of object's parts, compactness, symmetry, and so on) and to measure the relative position of different objects in terms of distance and direction.

Almost all known spatial extensions of standard SQL adopt a few metric operators, typically the computation of the *area* of a region, or the *distance* operator to formulate queries of the type: "Show all the cities at most 100 Km away from Rome." The latter is expressible in an SQL-like query as follows:

```
SELECT c2.name
FROM City as c1, City as c2
WHERE Distance (c1.location, c2.location)
      <=100km AND c1.name="Rome";
```

where the `Distance` operator has the meaning given in Table 1 and the locations refer to the centres of cities that are stored in the database.

Future spatial QLs, besides offering such quantitative operators, should offer *qualitative* operators as well. Recent results in Artificial Intelligence about qualitative spatial reasoning with distances and directions encourage the use of qualitative information as a viable alternative whenever quantitative information is not fully available or is not desired. Qualitative operators are essential to accommodate natural language queries which are common in real life.

For example, two contributions [CDFH97, Her94] have introduced flexible models for the qualitative description of orientation and distance relations, respectively. The relations are evaluated between a primary object and a reference object with respect to a *frame of reference*, which captures the inherent context dependency of such relations. The models describe different levels of granularity with a varying number of distance and orientation distinctions and the rules to mediate between them.

Within the formal framework defined in the paper [CDFH97], it becomes natural to pose queries like the following: "Display the cities located northeast of Rome and reachable quicky by car."

The latter query can be expressed in an SQL style as follows:

```
SELECT c2.name
FROM City as c1, City as c2
WHERE MediumDistance (c1.location,
   c2.location) AND c1.name="Rome"
AND northEast (c2, c1);
```

where `MediumDistance` is a qualitative operator that can be tought of as part of a set of qualitative distance operators made up of four distinctions like the following: {`close`, `medium`, `far`, `veryFar`}.

The major drawback of current models for qualitative distances and directions is that they approximate real objects as representative points. The extension of the available knowledge towards objects with shape and size is definitely urgent. A recent contribution in that sense is the paper by Goyal and Egenhofer [GE00].

References

[AMS98] G. Amato, G. Mainetto, and P. Savino. An approach to a content-based retrieval of multimedia data. *Multimedia Tools and Applications*, 7:9–36, 1998.

[C+93] Alfonso F. Cardenas et al. The knowledge-based object-oriented PICQUERY+ language. *IEEE Transactions on Knowledge and Data Engineering*, 5(4):644–657, 1993.

[CDF95] Eliseo Clementini and Paolino Di Felice. A comparison of methods for representing topological relationships. *Information Sciences*, 3:149–178, 1995.

[CDF96a] E. Clementini and P. Di Felice. An algebraic model for spatial objects with indeterminate boundaries. In P. Burrough and A. U. Frank, editors, *Geographic Objects with Indeterminate Boundaries*, GISDATA series, chapter 11, pages 155–169. Taylor & Francis, London, 1996.

[CDF96b] Eliseo Clementini and Paolino Di Felice. A model for representing topological relationships between complex geometric features in spatial databases. *Information Sciences*, 90(1–4):121–136, 1996.

[CDF97a] Eliseo Clementini and Paolino Di Felice. Approximate topological relations. *International Journal of Approximate Reasoning*, 16:173–204, 1997.

[CDF97b] Eliseo Clementini and Paolino Di Felice. A global framework for qualitative shape description. *GeoInformatica*, 1(1):11–27, 1997.

[CDF98] Eliseo Clementini and Paolino Di Felice. Topological invariants for lines. *IEEE Transactions on Knowledge and Data Engineering*, 10(1):38–54, 1998.

[CDFC95] Eliseo Clementini, Paolino Di Felice, and Gianluca Califano. Composite regions in topological queries. *Information Systems*, 20(7):579–594, 1995.

[CDFH97] Eliseo Clementini, Paolino Di Felice, and Daniel Hernández. Qualitative representation of positional information. *Artificial Intelligence*, 95:317–356, 1997.

[CDFK00] Eliseo Clementini, Paolino Di Felice, and Krzysztof Koperski. Mining multiple-level spatial association rules for objects with broad boundaries. *Data & Knowledge Engineering*, page to appear, 2000.

[CG96] A. G. Cohn and N. M. Gotts. Representing spatial vagueness: a mereological approach. In *Principles of Knowledge Representation and Reasoning: Proc 5th International Conference (KR96)*. Morgan Kaufmann, San Mateo, CA, 1996.

[CHCT98] W.W. Chu, C.C. Hsu, A.F. Cardenas, and R.K. Taira. Knowledge-based image retrieval with spatial and temporal constructs. *IEEE Transactions on Knowledge and Data Engineering*, 10(6):872–888, 1998.

[CRCB93] A. G. Cohn, D.A. Randell, Z. Cui, and B. Bennet. Qualitative spatial reasoning and representation. In N. Piera Carreté and M. G. Singh, editors, *Proceedings of the III IMACS International Workshop on Qualitative Reasoning and Decision Technologies—QUARDET'93—*, Barcelona, June 1993. CIMNE.

[EF91] Max J. Egenhofer and Robert Franzosa. Point-set topological spatial relations. *International Journal of Geographical Information Systems*, 5(2):161–174, 1991.

[Ege94] Max J. Egenhofer. Spatial SQL: A query and presentation language. *IEEE Transactions on Knowledge and Data Engineering*, 6(1):86–95, 1994.

[EH91] Max J. Egenhofer and John Herring. Categorizing binary topological relationships between regions, lines, and points in geographic databases. Technical report, University of Maine, Department of Surveying Engineering, 1991.

[EM95] Max J. Egenhofer and David M. Mark. Naive geography. In Andrew U. Frank and Werner Kuhn, editors, *Spatial Information Theory. A Theoretical Basis for GIS. International Conference, COSIT'95*, volume 988 of *Lecture Notes in Computer Science*, pages 1–15, Berlin, September 1995. Springer.

[ES97] M. Erwig and M. Schneider. Vague regions. In M. Scholl and A. Voisard, editors, *Advances in Spatial Databases - Fifth International Symposium, SSD '97*, volume 1262 of *Lecture Notes in Computer Science*, pages 298–320. Springer, Berlin, 1997.

[ES98] Max J. Egenhofer and A. Rashid B.M. Shariff. Metric details for natural-language spatial relations. *ACM Transactions on Information Systems*, 16(4):295–321, 1998.

[GE00] R.K. Goyal and M. J. Egenhofer. Cardinal directions between extended spatial objects. *IEEE Transactions on Knowledge and Data Engineering*, page to appear, 2000.

[GS95] Ralf H. Güting and Markus Schneider. Realm-based spatial data types: The ROSE algebra. *The VLDB Journal*, 4:100–143, 1995.

[Güt94] Ralf H. Güting. An introduction to spatial database systems. *The VLDB Journal*, 3(4):357–400, 1994.

[Her94] Daniel Hernández. *Qualitative Representation of Spatial Knowledge*, volume 804 of *Lecture Notes in Artificial Intelligence*. Springer, Berlin, 1994.

[PS94] D. Papadias and T. Sellis. Qualitative representation of spatial knowledge in two-dimensional space. *The VLDB Journal*, 3(4):479–516, 1994.

[S+99] S. Shekhar et al. Spatial databases: Accomplishments and research needs. *IEEE Transactions on Knowledge and Data Engineering*, 11(1), 1999.

[Sch96a] C. Schlieder. Qualitative shape representation. In P. Burrough and A. U. Frank, editors, *Geographic Objects with Indeterminate Boundaries*, GISDATA series, chapter 9, pages 123–140. Taylor & Francis, London, 1996.

[Sch96b] M. Schneider. Modelling spatial objects with undetermined boundaries using the realm/rose approach. In P. Burrough and A. U. Frank, editors, *Geographic Objects with Indeterminate Boundaries*, GISDATA series, chapter 10, pages 141–152. Taylor & Francis, London, 1996.

[The98] The OpenGIS Consortium. OpenGIS simple features specification for SQL. Technical report, http://www.opengis.org/techno/specs.htm, March 1998.

[Wor98] M. F. Worboys. Imprecision in finite resolution spatial data. *GeoInformatica*, 2(3), 1998.