Modelling Temporal Thematic Map Contents

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1 Introduction

In the last decade the database research community gave valuable results in modelling and retrieving spatial objects, in a temporal framework, e.g., [8] [9]. It is recognized that the representation and the management of GIS play a central part in data manipulation and querying. In fact, the incorporation of time in them may lead to consider new types of information for modelling those highly time dependent spatial data (e.g., temperatures, land use coverage, epidemic information related to a given area, etc.) and moving objects (e.g., entities that change the relative spatial position). The trend of temporal data modelling in GIS is moving from time-stamping layer (the snapshot models, [2]), attributes (space-time composites, [5]), and spatial objects to time-stamping events or processes that are mainly based on the concepts of time sequences [8]. Generally, in GIS the properties and the relations of geographic features are captured visually by maps. These maps are processed at one time and for one purpose. The class of maps that are developed to aid in decisions are called thematic maps. A thematic map is a map depicting selected kinds of information relating to one, or more specific themes, such as soil type, the incidence of a disease or land classification. Therefore, the manipulation of thematic maps for spatial query languages is an important task. In this paper, we will focus on temporal evolution of thematic maps in order to answer queries that refer to the time varying elements characterizing a thematic map, i.e., the geometric component of the map and the associated alphanumeric information. In this context, we first define through a *functional* approach (functional in the sense that it uses the function as mathematical structure) the notions of basic maps, thematic maps, and a set of operators for their manipulation. Then, we define a *space-time* composite functional approach by associating time to the above mentioned notions in order to model the temporal evolution of maps information, and we propose some new temporal operators. In our model, time can be conceptualized as instances (point-based) or intervals [1] [4]. To motivate our proposal, let us consider the query as follows: "Show distribution of the number of viral hepatitis-B cases in children below 10 years old in the Italian regions in the last century". For answering this query, the map model besides the spatial "where", and thematic "what", must support "when". Although many studies have been made in the context of spatio-temporal databases, current databases and systems related to them lack the ability to handle the temporal evolution of information in the thematic maps. In [7], for instance, efforts have been made to provide the designer of geographic databases with a query language extensible by a set of operations for static thematic map manipulation. In this work, the thematic map model is based on an object model and emphasizes the separation between map level and geometric level. The main idea is the association of the geometric operations with the data model through general constructs. However, in the proposed model the set constructor is used intensively inside ADT manipulation, which leads to a model that is difficult to understand. In [3], several operators on map which are based on a formal model of spatial partitions are introduced. Although, this work provides a theoretical framework for studying static maps, but the legend is described simply by mapping to a spatial partition a "label" type. Our approach overcomes the limitations of the above mentioned proposals by integrating both map, and geometric model into a unique functional model, and in addition defines by suitable functions the temporal variations of map contents. Furthermore, our approach is compatible with DBV version mechanism discussed in [6], which keeps track of different versions of the same geographic entity through time. The paper is structured as follows: Section 2 models general thematic maps called *static* thematic maps by a functional approach. Section 3 proposes a set of operators for manipulating the thematic maps depending on the number of operands in input, and user applications. Section 4 describes the definitions of temporal thematic map. Section 5 extends the operators defined in section 3 with time. Section 6 illustrates the proposed operators by some query examples, and finally, Section 7 concludes.

2 Functional Modelling

In this work, the geometry of the geographic features that we will refer to for describing all the following definitions and operators is region that is denoted by r. It is a subset of $\mathbb{R}^2$ and it is such that $\forall \epsilon > 0$, $\forall x \in r$, $\forall p \in \overline{r} = (\mathbb{R}^2 - r)$ it holds: $\mu(Ball(x,\varepsilon) \bigcap r) > 0 \wedge \mu(Ball(p,\varepsilon) \bigcap \overline{r}) > 0$, where μ is the Riemann's measure. Before introducing the concept of thematic map, we will define some basic notions including first of all the notion of basic maps. A *basic* map is a set of topographic data displayed in map form providing a frame of reference (i.e. the location data or position data) or contextual information to the user. Therefore, we give the following definitions for modelling the basic maps.

Definition 1 *A basic map M is a set of regions $M = \{r_1, \ldots, r_n\}$ with $\mu(r_i \bigcap r_j) = 0$ for $i \neq j$, and $i, j = 1, \ldots, n$.*

From the geometrical point of view, the above definition means that two different regions on a given basic map can be disjoint or they can intersect each other in a set of points or polyline.

Definition 2 *Let $f : r_1 \cup ... \cup r_n \rightarrow \mathbb{R}$ be a function. Let $I_i = [min_i, max_i)$ be a range of value where min_i and max_i represent, respectively, the relative minimum and maximum value. The function f maps each point x of a generic region r_i of a basic map to a value belonging to I_i. Formally, we have that $\forall x \in r_i$, $f(x) \in I_i$. This function satisfies the following condition: $\forall i, j \ (I_i \cap I_j = \emptyset) \vee (I_i = I_j)$*

Note that the above mentioned condition specifies that the function f is defined on a set of N_p disjoint intervals, where $N_p \leq N$. For sake of simplicity, assume that all intervals be consecutive. Then we have: $\bigcup_{q=1}^{N_p} [min_q; max_q) = [a, b)$ where $a = min_1, b = max_{N_p}$.

Definition 3 *Let M and $I = \{I_1, \ldots, I_{N_p}\}$ be, respectively, a basic map and a set of ranges. We define the function $\varphi : M \rightarrow I$ that map each region to a range.*

Note that generally, the function φ is not invertible, this means that to a unique range more regions can be associated.

Definition 4 *Let $\psi : I \rightarrow C$ be a invertible function that map the ranges to the elements belonging to a set $C = \{c_1, \ldots, c_k\}$. This set can represent: a) The set of RGB colours; b) A finite set of pattern (bitmap) that are used to fill different areas; c) a finite set of strings. Then, we define the composed function $\Gamma = \psi \circ \varphi$ that maps each region of basic map to an element belonging to C . We will call it correspondence function.*

Definition 5 *The legend is defined as the following pair: $\lambda = \langle name, \psi^{-1} \rangle$, where name is a string which indicates the name of the legend.*

For instance, the *name* could be "incidence of cholera". By using the above definitions, we can formalize thematic map as follows:

Definition 6 *A thematic map (called θ) is a quadruple: $\theta = \Big\langle Name, \lambda, M, \Gamma \Big\rangle$, where $Name$ is a string that represents the name of thematic map.*

Note that between the name of thematic map and the name of the associated legend often a relationship exists; for instance, $Name =$ "incidence of cholera in Africa" and $\lambda.name =$ "incidence of cholera" . The thematic maps will be also denoted by a shortened form as follows: $\Big\langle Name, \lambda, \{\{r_1, c_1\}, \ldots, \{r_k, c_k\}\}\Big\rangle$. For sake of notational simplicity, we will omit the name of the thematic maps where it will not be essential in the definitions which follow. We will point out that the concept of thematic map can be generalized by allowing that the set I is defined as a generic finite set, e. g., as a set of strings. For instance, a geographic administrative map is also a thematic map assuming that I is a numerable set strings (the regions names) and $C = I$. All the definitions and the set of operators which we will give in the following are valid for the generalization of thematic maps, except for some operators which will use the properties of the real numbers. Note that, if in the definition of thematic map we assume that M is a set of polylines and μ represents the length function, then we can apply formally the above definitions and the following operators. In this way, we obtain another kind of thematic map: the *linear* thematic map. This type of map could be useful for the description, for example, of the pollution of a river subdivided in a set of lines, for each of which the percentage of the pollution is given.

3 Set of Operators

In the following subsections, we propose a set of operators that can have one, two or more thematic maps as operands in input and output. Furthermore, a set of operators for user-oriented applications is proposed. They will refer to the generic set of thematic maps that is represented by the symbol Θ.

3.1 One operand

In It is related to testing if a given region r is present in a thematic map $\theta \in \Theta$. The boolean operator In gives in output the result $true$ if the region r is present in the basic map of $\theta = \Big\langle Name, \lambda, \{\{r_1, c_1\}, \ldots, \{r_k, c_k\}\}\Big\rangle$, $false$ otherwise. Formally: $In(r, \theta) = \vee_{i=1}^{k}(r = r_i)$.

AtLeast Let $\theta \in \Theta$ be a thematic map and let m be a real number. The operation $AtLeast(m, \theta)$ gives as result the set of regions in the thematic map for which the associated thematic value is greater than or equal to m. Formally we have: $AtLeast(m, \theta) = \Big\{r_i \Big| (\psi^{-1} \to [p, q)) \wedge (p \geq m))\Big\}$.

AtMax This operator is the dual of *AtLeast* operator. It gives the set of regions for which the associated thematic value is less than or equal to a real value m: $AtMax(m, \theta) = \Big\{r_i \Big| (\psi^{-1} \to [p, q)) \wedge (q \leq m))\Big\}$.

Between Let $\theta \in \Theta$ be a thematic map and let $[a, b]$ be a range. This operator gives the set of regions in θ for which the associated thematic value belongs to range $[a, b]$, then formally: $Between([a, b], \theta) = AtLeast(a, \theta) \cap AtMax(b, \theta)$.

Selection Let $\theta = \Big\langle Name, \lambda, \{\{r_1, c_1\}, \ldots, \{\mathrm{r}_k, c_k\}\}\Big\rangle$ be a thematic map and let $x = [a, b)$ be an interval on real axis. This operation gives the regions of map whose thematic values coincide with interval x: $Selection(x, \theta) = \Big\{r_i \Big| x = \psi^{-1}(c_i)\Big\}$.

Fusion This operator allows to obtain a new thematic map realizing the geometric union of the regions of a map which have the same thematic value. It is defined as follows: $\mathrm{Fusion}(\theta) = \Big\langle Name, \lambda, \{\varrho, c | (\varrho = \cup Selection(\psi^{-1}(c), \theta)) \wedge (\varrho \neq \emptyset) \wedge (c \in C)\}\Big\rangle$. The resulting map is defined by a set of pairs $\langle \varrho, c\rangle$, where ϱ is the geometric union of the regions which have a common thematic value, and c belongs to the set defined in Definition 4.

3.2 More operands

Intersection between a thematic map and a region Let $\theta = \Big\langle \lambda, \{\{r_1, c_1\}, \ldots, \{r_k, c_k\}\}\Big\rangle$ be a thematic map and let R be a region. The intersection yields a map defined by the same legend of θ and a set of regions that is the result of the intersection of the regions belonging to θ with R. Formally: $Intersection(\theta, R) = \Big\langle \lambda, \{r_1 \cap R, c_1, \ldots, r_k \cap R, c_k\} \Big\rangle$.

Union of thematic maps Let $\theta_1 = \langle \lambda_1, \Gamma_1, M_1 \rangle$ and $\theta_2 = \langle \lambda_2, \Gamma_2, M_2 \rangle$ be thematic maps, thus their basic maps are: M_1 and M_2. Two conditions are needed for applying the union operator on them. The first condition is that the thematic maps must share the same legend. The second condition is that the basic maps must be disjoint or, otherwise, the regions belonging to $M_1 \cap M_2$ have the same thematic values. Summarizing, in order to apply this operator the following condition must hold: $(\lambda_1 = \lambda_2) \wedge \Big((M_1 \cap M_2 = \emptyset) \vee (\forall u \in M_1 \cap M_2, \Gamma_1(u) = \Gamma_2(u)) \Big)$. If this condition is satisfied, then by defining the following function: $\Gamma(x) = \Gamma_1(x) \text{if } x \in M_1, \Gamma_2(x) \text{if } x \in M_2$, which we will call *correspondence function*, the union of θ_1, θ_2 is defined as follows: $Union(\theta_1, \theta_2) = \langle \lambda, \Gamma, M_1 \cup M_2 \rangle$.

Intersection between a thematic map and a geographic map Let $M = \{R\}$ be a geographic map defined by a unique region. Let $\theta = \Big\langle \lambda, \{\{r_1, c_1\}, \ldots, \{r_k, c_k\}\} \Big\rangle$ be a thematic map. The intersection between these maps is: $Intersection(\theta, \{R\}) = \Big\langle \lambda, \{\{r_1 \cap R, c_1\}, \ldots, \{r_k \cap R, c_k\}\} \Big\rangle$. When $M = \{r_1, \ldots, r_n\}$, the intersection operation is given by: $Intersection(\theta, M) = Union\Big(Intersection(\theta, \{r_1\}), \ldots, Intersection(\theta, \{r_n\}) \Big)$. In the Figure 1, an example of the application of this operation is shown.

3.3 User-Oriented Applications

In this section, we define a new operator, which we call *Picking*, for the interaction with a Thematic Map and we formalize with our functional approach two other operators, windowing and clipping, which were defined previously in [7].

Picking Let $\theta = \langle \lambda, M, \Gamma \rangle$ be a thematic map and let W be a selected rectangular or circular area

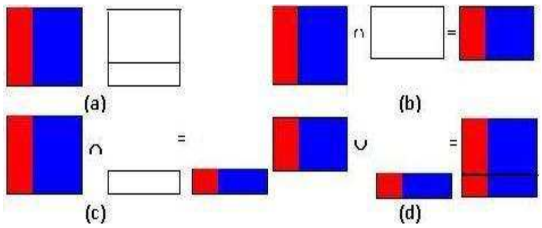

Figure 1: Illustration of Intersection between a thematic map and a geographic map.

on the screen selected through the mouse. The application of this operation produces a new thematic map containing all regions of θ which are totally included in W. The semantic of this operation is defined as follows: $Pick(\theta, W) = \Big\langle Name, \lambda, \{\langle \varrho, c \rangle | (\varrho \in M) \wedge (\varrho \cap W = \varrho) \wedge (c = \Gamma(\varrho))\} \Big\rangle$, where, in the pairs $\langle \varrho, c \rangle$, ϱ is a region of the thematic map, which belongs totally to W, and $c \in C$.

Windowing Let $\theta = \langle \lambda, M, \Gamma \rangle$ be a thematic map and let W be a selected rectangular or circular area on the screen selected through the mouse. The application of this operation produces a new thematic map containing all regions of θ which have a not null bi-dimensional intersection with W. The semantic of this operation is defined as follows: $Wind(\theta, W) = \Big\langle Name, \lambda, \{\langle \varrho, c \rangle | (\varrho \in M) \wedge (\varrho^\circ \cap W \neq \emptyset) \wedge (c = \Gamma(\varrho))\} \Big\rangle$, where, in the pairs $\langle \varrho, c \rangle$, ϱ is a region of the thematic map, whose interior (ϱ°) has a not null intersection with the selected window, and $c \in C$.

Clipping Let $\theta \in \Theta$ be a thematic map and let W be a selected rectangular or circular area on the screen, that is selected through the mouse. This operation gives as result the regions that are enclosed in the selected area. It is a particular case of the intersection operation between a thematic map and a region: $Clip(\theta, W) = Intersection(\theta, \{W\})$. In Figure 2, an example of application of Pick, Wind and Clip operators is shown.

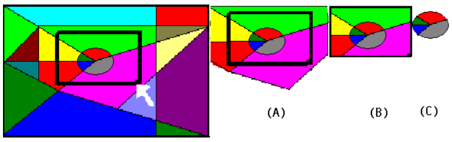

Figure 2: Results of application of windowing (A), clipping (B) and picking (C) operators.

4 Temporal Thematic Maps

Considering the definitions given in the previous sections, the concept of time becomes meaningfully important in the context of thematic maps. In fact, in a thematic map either the geographic part or the thematic part need some temporal elaboration. For instance, let us consider a thematic map that represents the "distribution of annual cholera cases in Europe " and the temporal interval $[1/1/1989, 1/1/1999)$. In this interval of time, we can consider that the temporal evolution of theme of this map is constant but in the context of geographic map and then the underlying subdivision of the boundary of countries there has been some well known variations. Now, let us consider the thematic map that represents "distribution of the number of viral hepatitis B cases in children below 10 years old in the Italian regions" and let the temporal interval be $[1/1/1985, 1/1/1995)$. In this period, thanks to the HBV vaccination policy, the annual number of hepatitis B cases (theme) has significantly decreased, but the geographic part (Italian regions) did not undergo variations. If we consider the thematic map of the distribution of this disease in the period $[1/1/1985, 1/1/1995)$ in Europe, both thematic and geographic parts have assumed temporal variations.
Generally, the temporal dynamics of theme and geographic one are independent, and for this reason their evolutions are considered to be asynchronous. Starting from the formalization of thematic maps introduced in the section 2, and considering the following assumptions: a)the legend and the name of map do not change in time; b) the basic map changes; c) the function φ and then Γ change in time and d) the variations are constant in time intervals (this means that in the considered interval there is no continuous variations), we give the definition of temporal (or temporized) thematic map. Note that, in the following we will use the terms temporal and temporized interchangeably.

Definition 7 *A temporized thematic map is defined by the following triple:* $\langle Name, \lambda, \{(\tau^1, \langle M^1, \Gamma^1\rangle) , \ldots, (\tau^\omega, \langle M^\omega, \Gamma^\omega\rangle)\}\rangle$*, where* $\tau^1, \ldots, \tau^\omega$ *represents a series of time intervals, in each of which we consider that the evolution of the thematic and geographic parts of the given map are constant. In a generic interval* τ^i *, the considered constant thematic map is defined by basic map* $M_i = \{r_1^i, \ldots, r_{N_i}^i\}$ *and the function* $\Gamma\Big(\{r_1^i, \ldots, r_{N_i}^i\}\Big) = \{c_1^i, \ldots, c_{N_i}^i\}$*.*

When the indication of the legend and the Name is not essential, a temporal thematic map can be simply represented by a set of couples $\{\langle\tau^1, \theta^1\rangle, \ldots, \langle\tau^\omega, \theta^\omega\rangle\}$, each of which represents time interval and a constant thematic map. All these thematic maps share the same legend, and the time ranges are disjoint and ordered. For sake of brevity, we will indicate the set of all Temporized Thematic Maps by the symbol: $\widehat{\Theta}$. Note that a static thematic map $\langle Name, \lambda, M, \Gamma\rangle$ can be considered as special case of a temporized thematic map and can be represented as follows:$\langle Name, \lambda, ([t_o, now], \langle M, \Gamma\rangle)\rangle$, where t_o is the initial time which is considered in the system and *now* refers to the present time.

5 Operators

For the Temporized Thematic Maps, we extend the set of operators defined in section 3, and we define new others for modelling some basic interactions with time or time intervals. In the following, we will refer to $\widehat{\theta}$ for indicating a generic element of $\widehat{\Theta}$. The temporized unary operators are defined in terms of the static unary operators. They are listed in the Table 1, and for each of them the formalization and description are given. We define the following operators with two or more operands.
Union of temporized thematic maps The application of this operator, as in the static case, will be possible

Op.s	Formalization	Description
In	$In\left(\varrho,\widehat{\theta}\right) = (\exists i, In(\varrho,\theta_i))$	It yields true value if at least for one element of temporized thematic maps, the result of the "static" In operator is True.
$Selection$	$Selection\left(x,\widehat{\theta}\right) = \bigcup_{j=1}^{\omega}\left\{\langle\tau_i, Selection(x,\theta_i)\rangle\right\}$	It applies the static version of Selection to each $\theta_1,\ldots,\theta_\omega$ and the result is a temporized thematic map.
$Fusion$	$Fusion(\widehat{\theta}) = \bigcup_{j=1}^{\omega}\left\{\langle\tau_i, Fusion(\theta_i)\rangle\right\}$	See the description of *Selection*.
$AtLeast$	$AtLeast(m,\widehat{\theta}) = \bigcup_{j=1}^{\omega}\left\{\langle\tau_i, AtLeast(m,\theta_i)\rangle\right\}$	See the description of *Selection*.
$AtMax$	$AtMax(m,\widehat{\theta}) = \bigcup_{j=1}^{\omega}\left\{\langle\tau_i, AtMax(m,\theta_i)\rangle\right\}$	See the description of *Selection*.
$Between$	$Between\left([m_1,m_2],\widehat{\theta}\right) = \bigcup_{j=1}^{\omega}\left\{\langle\tau_i, Between([m_1,m_2],\theta_i)\rangle\right\}$	See the description of *Selection*.

Table 1: Operators with one temporized thematic map operand.

if and only if there is unique legend shared by all the operands. First of all, we will consider the simple case of the union of two very simple elements of $\widehat{\Theta}$: $\widehat{\theta}_a = \{\tau_a,\theta_a\}, \widehat{\theta}_b = \{\tau_b,\theta_b\}$. If $\tau := \tau_a \cap \tau_b = \emptyset$, then their union is defined as: $Union(\widehat{\theta}_a,\widehat{\theta}_b) = \{\langle\tau_a,\theta_a\rangle,\langle\tau_b,\theta_b\rangle\} = \{\langle\tau_a,\theta_a\rangle\} \cap \{\langle\tau_b,\theta_b\rangle\}$. If $\tau \neq \emptyset$, this operator will be defined as follows: $Union(\widehat{\theta}_a,\widehat{\theta}_b) = \{\langle\tau_a-\tau,\theta_a\rangle,\langle\tau,Union(\theta_a,\theta_b)\rangle,\langle\tau_b-\tau,\theta_b\rangle\} \neq \{\langle\tau_a,\theta_a\rangle\} \cap \{\langle\tau_b,\theta_b\rangle\}$.
For generalizing the above definition, let $\widehat{\theta} = \{\langle\tau_1,\theta_1\rangle,\langle\tau_P,\theta_P\rangle\}$ and $\widehat{\theta}^* = \{\langle\tau_1^*,\theta_1^*\rangle,\langle\tau_Q^*,\theta_Q^*\rangle\}$ be elements of $\widehat{\Theta}$. Their union is: Union$(\widehat{\theta},\widehat{\theta}^*) = \bigcup_{i=1,j=1}^{i=P,j=Q} Union(\{\langle\tau_i,\theta_i\rangle\},\{\langle\tau_j^*,\theta_j^*\rangle\})$.

Intersection of a temporized thematic map with a region This operator is an extension of the previous one defined for static thematic Maps. It will be applied to obtain not only the intersection between a temporized thematic map and a region or map, but also a more time-dependent entities which are temporized region and temporized map. The result of this operator will be a temporized thematic map. In this paper, we will give a first rough (but complete, for the definition of these operators) definition of these entities, which are part of an our work in progress on a general model for temporized geographic objects. The intersection of a temporized thematic map with a region R can be defined as follows: Intersection$(\widehat{\theta},R) = \bigcup_{i=1}^{\omega} Union(\{\langle\tau_i, Intersection(\theta_i,R)\rangle\})$.

In this definition R represents a generic region without any temporal variation. Since, the spatial configuration of a region can change in time, we define a new entity which we call temporized region ϱ. It consists of a set of pairs formed by a time interval and a static region: $\varrho = \left\{\langle\sigma_1,\rho_1\rangle,\ldots,\langle\sigma_n,\rho_n\rangle\right\}$ where $\sigma_1,\ldots,\sigma_n$ are consecutive disjoint temporal intervals and $\rho_1,\ldots,\rho_n$ represents different spatial configurations of a given region in each interval. Many examples of temporized regions can be mentioned in various fields such as ecology, history (e.g. Italy became an independent country in 1861 and along time the relative boundary is changed, respectively, in 1866, 1870, 1890, 1912, 1918, 1924, 1939, 1945, 1947 and 1954), etc. Then, the intersection of a temporized region with a temporized thematic map will be : $Intersection(\widehat{\theta},\varrho) = \bigcup_{(i,j)\in\{(1,1),\ldots,(\omega,n)\}} Union(\{\langle\tau_i\cap\sigma_j, Intersection\ (\theta_i,\rho_j)\rangle\})$. In the same way by defining a temporized map as $M = \left\{\langle\sigma_1,M_1\rangle,\ldots,\langle\sigma_n,M_n\rangle\right\}$ it is possible to define the intersection of a temporized thematic map with a temporized map as follows: $Intersection(\widehat{\theta},M) = \bigcup_{(i,j)\in\{(1,1),\ldots,(\omega,n)\}} Union(\{\langle\tau_i\cap\sigma_j, Intersection$

$(\theta_i, M_j)\rangle\})$.

Example 1 Let us consider a temporized thematic map whose name is "Diffusion of flu in Europe between the years 1900 and 1999". A query on this map that invoke the above operator is "Get the map of the flu in Mediterranean countries of Europe in the period $[1/1/1991, 1/1/1992)$".

TSlice Let $\widehat{\theta}$ an element of $\widehat{\Theta}$ and let t an instant of time. This binary operator gives the static thematic map belonging to $\widehat{\Theta}$ at the time t: $\text{TSlice}(\widehat{\theta}, t) = \theta_i, \text{if } \exists i | t \in \tau_i, \emptyset$ otherwise.

$ExtendedTSliceLet\widehat{\theta}$ be an element of $\widehat{\Theta}$ and let τ be a time interval. This binary operator gives the temporal thematic map which contains all the thematic maps belonging to $\widehat{\Theta}$ during the time interval τ: $ExtendedTSlice(\widehat{\theta}, \tau) = \bigcup_{j=1}^{\omega} \left\{ \langle \tau_i \cap \tau, \theta_i \rangle \right\})$. For instance, given the temporized thematic map of Example 1, "get the map of the flu in Europe in the period $[1/1/1919, 1/1/1920)$".

Before defining the operators which model the interaction of an user with the elements of $\widehat{\Theta}$, it is, of course, mandatory to define how to visualize a temporized thematic map. We identified two different visualization modalities: a) visualization of an unique legend and a series of static thematic maps with the relative temporal intervals and b) visualization of the legend and the representation of a sequence of static thematic map through animation that yields the evolution of thematic map objects during each temporal interval. Let $\widehat{\theta} = \{\langle \tau_1, \theta_1 \rangle, \ldots, \langle \tau_\omega, \theta_\omega \rangle\}$ be the element of $\widehat{\Theta}$ on which the user will interact. At first, we shall examine the user interaction when the first way of visualization is chosen. For each of the $\theta_1, \ldots, \theta_\omega$ visible on the screen, the user will have to select a region trough the mouse. So, at the end of this process, a set of regions $w_1, \ldots, w_\omega$ is selected. Since a temporal interval is linked to each of the $\theta_1, \ldots, \theta_\omega$, we define the following temporized region which is the implicit result of the selections made by the user on the screen: $W = \{\langle \tau_1, w_1 \rangle, \ldots, \langle \tau_\omega, w_\omega \rangle\}$. If in some of the component thematic maps the user does not perform a selection, then we assume the values of the corresponding regions be equal to the empty set. If the chosen visualization mode is the animation, the user can perform only one region selection and this selection has to be performed before the starting of the animation. Also in this case we may define an implicitly selected temporized region $W = \{\langle \tau_1, w_1 \rangle, \ldots, \langle \tau_\omega, w_\omega \rangle\}$, where this constraint holds: $w_1 = w_2 = \cdots = w_\omega$. Therefore, in both the above visualization modalities, we can define the three interaction operators $Pick$, $Wind$ and $Clip$:

$\text{Pick}(\widehat{\theta}, W) = \bigcup_{i=1}^{\omega} Union(\{\langle \tau_i, Pick(\theta_i, w_i) \rangle\})$;

$\text{Wind}(\widehat{\theta}, W) = \bigcup_{i=1}^{\omega} Union(\{\langle \tau_i, Wind(\theta_i, w_i) \rangle\})$;

$\text{Clip}(\widehat{\theta}, W) = \bigcup_{i=1}^{\omega} Union(\{\langle \tau_i, Clip(\theta_i, w_i) \rangle\})$.

6 Thematic map querying

In this section, we illustrate the applicability of the proposed set of operators defined for static and temporal thematic maps to data queries by means of the following query examples. The queries are expressed by an OQL-like language.

Query 1: *"Find all thematic maps to which the nation Italy belongs"*. This query can be expressed as follows:

SELECT tm FROM tm IN ThematicMaps, n IN NATIONS WHERE n.name="Italy" AND In(n,tm)

Note that in this example both IN operator of OQL language and In operator defined for thematic maps are used with different semantics.

Query 2:*"Find the EU nations in which the annual incidence of asthma is greater than* 500 *cases per millions"*.

SELECT n.name FROM n IN NATION, tm IN ThematicMaps WHERE tm.name="annual inciden ce of asthma in EU" AND n in AtLeast(500,tm)

Query 3:*Find the Italian regions in which the percent incidence of asthma is between* 5% *and* 10% *of the populations"*.

SELECT r FROM r IN REGIONS, tm IN ThematicMaps WHERE

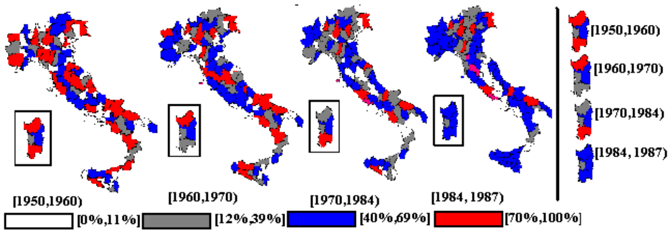

Figure 3: Application of Clip operation: selection of 4 rectangular windows (left), and results (right).

tm.name=”Distribution of asthma in Italian regions” AND r in Between([5,10],tm)
In this query REGIONS indicates the italian administrative subdivision. The above queries refer to the static thematic maps. In the following, we will give some query examples in which temporal thematic maps (TTMaps) are involved.
Query 4:*”Get the map of the flu in South Europe in* 1900 -1999*”.*
SELECT Intersection(ttm,seu) FROM ttm IN TTMaps, seu IN TemporizedMaps WHERE ttm.name=”Diffusion of flu in Europe in 1900-1999” AND seu.name =”South Europe”
Query 5:*”Get the map of the flu in EU in the period* $[1/1/1919, 1/1/1920)$*”.*
SELECT ExtendedTSlice(ttm,[1/1/1919, 1/1/1920)) FROM ttm IN TTMaps WHERE ttm.name=”Diffusion of flu in EU in 1900-1999”

7 Conclusions

In this paper, we have presented a functional approach enable to model the temporal evolution of map contents in a temporal interval. Because our model is based on mathematical functions, we can overcome the limitations of any underlying well known formal models (e.g., relational or object-oriented) for describing all elements characterizing a thematic map. We shown the power of our model on answering queries invoking temporal information through very simple query formulation.

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