Yet Another Note on Minimal Covers

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In [Atk88] Atkins corrects a widely spread error in the algorithm for finding a minimal cover for a given set of functional dependencies. The erroneous form of the algorithm has been presented in [Sal86,StW83,Ull82,Yan88]. Unfortunately, though, there is an error also in the corrected algorithm.

Atkins proposed the following algorithm for determining a minimal cover for a given set of functional dependencies F.

ALGORITHM 1 ([Atk88])

1. Ensure that every functional dependency has a single attribute right hand side.

2. Consider each dependency $X \rightarrow A$ in some order. If Z is a subset of X such that F is contained in the closure of $(F \setminus \{X \rightarrow A\}) \cup \{Z \rightarrow A\}$, then immediately replace $X \rightarrow A$ by $Z \rightarrow A$ in F. This step continues until no left side of any dependency in F can be reduced.

3. Consider the dependencies in F in some order. If $X \rightarrow A$ is in F and $X \rightarrow A$ is in closure of $(F \setminus \{X \rightarrow A\})$, then immediately remove $X \rightarrow A$ from F. This step continues until every dependency in F has been tested.

To point out the error we use an example. Consider relation scheme $R=(\{A,B,C\}, \{AB \rightarrow C\})$. There is only one functional dependency $AB \rightarrow C$ in the scheme and it has a single right hand side, so steps 1 and 3 of algorithm 1 do not alter the set of functional dependencies. In step 2 of algorithm 1 we could replace $AB \rightarrow C$ by $A \rightarrow C$, since using the augmentation axiom we get $\{A \rightarrow C\} \models AB \rightarrow C$. What results is not a cover of the set of initial functional dependencies, since $\{AB \rightarrow C\} \not\models A \rightarrow C$.

To maintain an equivalent set of functional dependencies it is not sufficient in step 2 to test if F is contained in the closure of $(F \setminus \{X \rightarrow A\}) \cup \{Z \rightarrow A\}$, one should also test whether $(F \setminus \{X \rightarrow A\}) \cup \{Z \rightarrow A\}$ is contained in the closure of F.

We point out that an algorithm which implements essentially the same idea as the corrected version of algorithm 1 can be found in Maier's book [Mai83, 74-77]. The correctness of that algorithm is also shown in [Mai83].

References

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